Quantum and nuclear physics (HL) 12

12.1 The interaction of matter with radiation

This section deals with an array of new phenomena. The photoelectric effect and the spectra of atoms were unsolved problems in physics for the entire second half of the 19th century. Their solution paved the way for quantum theory, with its own array of unusual concepts and phenomena such as the wavefunction, the uncertainty principle and tunnelling.

Learning objectives

- Understand the nature of photons and why they were needed to explain experimental results.
- Discuss the photoelectric effect.
- Understand the concept of matter waves.
- Solve problems involving pair production and pair annihilation.
- Understand the consequences of angular momentum quantisation in the Bohr model.
- Understand the concept of the wavefunction.
- Work with the Heisenberg uncertainty principle.
- Qualitatively understand barrier tunnelling and the factors affecting the tunnelling probability.

Photons and light

Light is said to be an **electromagnetic wave** consisting of oscillating electric and magnetic fields. This was Maxwell's great discovery in the 19th century. The wave has some wavelength λ and a frequency f and, as with all waves, the wave speed c is given by:

$$c = f\lambda$$

where in this case the wave speed is the speed of light.

Through Maxwell's theory, complex phenomena such as diffraction, interference, polarisation and others could be understood. The successful application of Maxwell's theory meant that light was definitely and without any doubt a wave. It therefore came as a shock that, in a phenomenon known as the photoelectric effect, light did not behave as a wave should. (We shall look at this phenomenon in more detail in the next subsection.)

As we will see, Einstein suggested that light should be thought of as a collection of quanta, or bundles of energy. Each **quantum** or bundle of light has energy E given by $E = hf$, where f is the frequency of the light and h is Planck's constant. A beam of light of frequency f is now to be thought of as a very large number of these quanta moving at the speed of light. The total energy of the beam is then the product of hf (the energy of one quantum) times N the number of quanta in the beam. The energy of the beam is therefore an integral multiple of the basic unit hf. No amount of energy less than hf would ever be found in the beam. These quanta have definite energy and are localised in space; this means that they behave as particles. But the theory of relativity states that if a particle moves at the speed of light it has to have zero mass. So this quantum of light, which came to be known as the **photon**, is a particle with zero mass and zero electric charge.

In Topic **7** we saw that a photon can be created when an atom makes a transition from a high to a lower energy. Its energy is the energy difference of the two levels. A photon can also be absorbed by an atom. An atom in a low energy state can absorb a photon of just the right energy and make a transition to a higher energy level. When we look at the light from a light bulb we see a continuous emission of light. But if we could slow down the process by a few billion times, the continuity in the emission of light would stop. We would see different spots on the filament emit tiny flashes of light (photons) at random interval of time; the spots on the filament would be on (emitting) and off (not emitting) randomly. The discreteness of energy we talked about in Topic **7** would surface again.

In Einstein's theory of special relativity the total energy E, the momentum p and the mass m of a particle are related according to:

$$E^2 = p^2c^2 + m^2c^4$$

The mass of the photon is zero, so $E = pc$. The photon therefore has momentum $p = \frac{E}{c}$. (This implies that the conventional Newtonian formula for momentum, $p = mv$, does not apply to particles with zero mass.) So the momentum of the photon is:

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$$

Exam tip
Remember the basic formula from waves: $c = f\lambda$.

Worked examples

12.1 Estimate how many photons of wavelength 5.0×10^{-7} m are emitted per second by a 60 W lamp, assuming that 1% of the energy of the lamp goes into photons of this wavelength.

Let there be N photons per second emitted.

Then the energy they carry is $\frac{Nhc}{\lambda}$ in one second.

This has to be 1% of 60 J, that is 0.60 J.

So: $\frac{Nhc}{\lambda} = 0.60$

$$N = \frac{0.60\lambda}{hc}$$

$$N = \frac{0.60 \times 5.0 \times 10^{-7}}{6.63 \times 10^{-34} \times 3.0 \times 10^{8}}$$

$\Rightarrow$ $N = 1.5 \times 10^{18}$ photons per second.

12.2 All the photons from Worked example **12.1** are incident normally on a mirror of area 0.5 m^2 and are reflected by it. Estimate the pressure these photons exert on the mirror.

Each photon has a momentum of $\frac{E}{c}$ or $\frac{h}{\lambda}$

The momentum **change** upon reflection is $2\frac{h}{\lambda}$ (momentum is a vector!).

Since there are N such reflections per second, the force F on the mirror is:

$$F = 2N\frac{h}{\lambda}$$

$$F = 2 \times 1.5 \times 10^{18} \times \frac{6.63 \times 10^{-34}}{5.0 \times 10^{-7}}$$

$$\Rightarrow \quad F = 4.0 \times 10^{-9}\,\mathrm{N}$$

The pressure is thus:

$$\frac{F}{A} = 8.0 \times 10^{-9}\,\mathrm{N\,m^{-2}}$$

(Note that if the photons were absorbed rather than reflected, the pressure would be half that obtained here.)

The photoelectric effect

The **photoelectric effect** is the phenomenon in which light (or other forms of electromagnetic radiation) incident on a metallic surface causes electrons to be emitted from the surface.

To investigate the facts about the photoelectric effect, apparatus like the one in Figure **12.1** may be used.

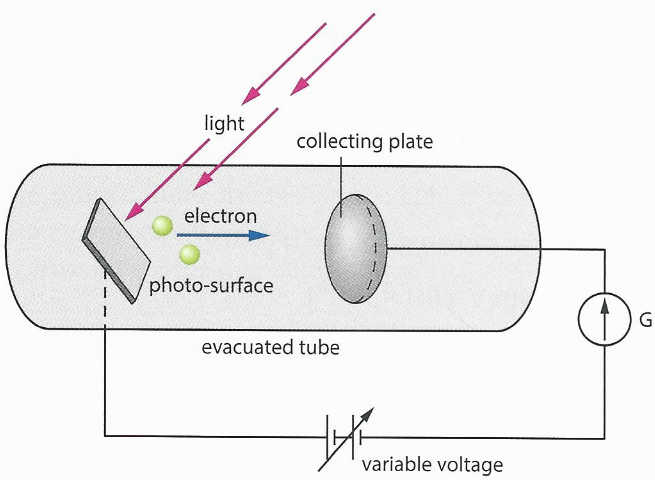


Figure 12.1 Apparatus for investigating the photoelectric effect. The variable voltage decelerates the emitted electrons and eventually stops them.

It consists of an evacuated tube, inside which is the **photo-surface** (the metallic surface that light is incident on). Light passes through an opening in the tube and falls on the photo-surface, which emits electrons. Some of the emitted electrons arrive at the collecting plate. The photo-surface and the collecting plate are part of a circuit as shown. Those electrons that

Exam tip

The stopping voltage is strictly negative but we work with its magnitude.

It is very important to understand that the stopping voltage gives the **maximum** kinetic energy of the emitted electrons.

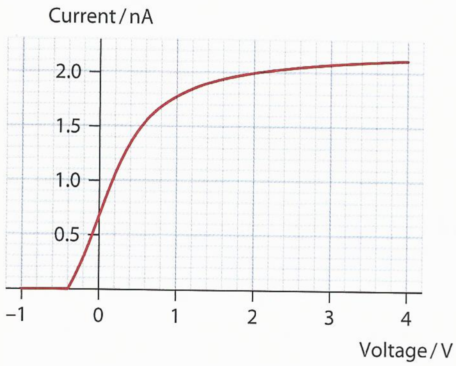


Figure 12.2 When the collecting plate is connected to the negative terminal of the power supply, there is a voltage at which the current becomes zero (V_s).

make it to the collecting plate complete the circuit and so we have an electric current that is recorded by the sensitive galvanometer.

Notice that in Figure **12.1** the negative terminal of the variable power supply is connected to the collecting plate. This means that the collecting plate actually repels the emitted electrons. Only the very energetic electrons will make it to the plate. As the magnitude of the voltage is increased (i.e. made more negative) fewer and fewer electrons make it to the plate; eventually no electron will arrive there and at that point the current becomes zero. The voltage at which the current becomes zero is called the **stopping voltage**, V_s. Its significance is that the maximum kinetic energy of the emitted electrons must be eV_s. We see this as follows: let the maximum kinetic energy of the electrons be E_{max} as they leave the photo-surface; the work done in moving an electron from the photo-surface to the collecting plate is eV_s. From mechanics we know that the work done is the change in the kinetic energy of the electron. So:

$$eV_s = E_{max}$$

We now connect the positive terminal of the power supply to the collecting plate. The electrons are now attracted to the collecting plate and the current increases. As the voltage is increased even more the current saturates, i.e. it approaches a constant value. This is because the collecting plate is so positive that it attracts every single emitted electron (even those that were not directed at the collected plate). So we have a current–voltage graph like the one in Figure **12.2**.

Worked example

12.3 Using the graph of Figure **12.2** determine:

a the stopping voltage

b the maximum energy of the emitted electrons

c the maximum speed of the emitted electrons.

a The current becomes zero when the voltage is –0.40 V so the stopping voltage is 0.40 V.

b The maximum kinetic energy of the emitted electrons is $0.40\,\text{eV} = 6.4 \times 10^{-20}\,\text{J}$.

c From $E_{max} = \frac{1}{2}mv^2$, we find $v = \sqrt{\frac{2E_{max}}{m}}$, giving:

$$v = \sqrt{\frac{2 \times 6.4 \times 10^{-20}}{9.1 \times 10^{-31}}} = 3.8 \times 10^5\,\text{m s}^{-1}$$

The results of this experiment reveal two immediate surprises: the first is that changing the intensity of the light does not affect the stopping voltage! Light from a candle and light from an airport searchlight give the same stopping voltage. Figure **12.3** shows that the stopping voltage for weak light (thin line) and intense light (thick line) are the same.

The stopping voltage is independent of the intensity of the light source.

The second surprise is that the stopping voltage depends on the frequency of the light. The higher the frequency, the higher the magnitude of the stopping voltage. This is shown in Figure **12.4**: the violet curve corresponds to violet light and the green curve to green light of lower frequency. The stopping voltages are 0.40 V for green and 1.0 V for violet.

If we plot the kinetic energy of the electrons (which equals eV_s) versus frequency, we find a straight line as shown in Figure **12.5a**.

The puzzling feature of this graph is that there exists a frequency, called the critical (or threshold) frequency f_c, such that no electrons at all are emitted if the frequency of the light source is less than f_c. This is true even if very intense light is allowed to fall on the photo-surface. When the experiment is repeated with a different photo-surface and the kinetic energy of the electrons is plotted versus frequency, a line parallel to the first is obtained, as shown in Figure **12.5b**.

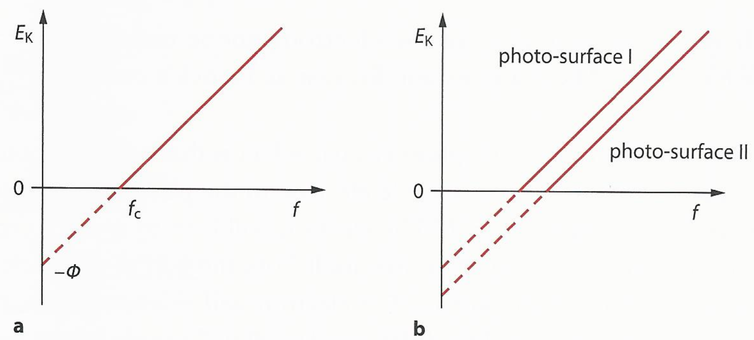


Figure 12.5 **a** The graph of kinetic energy versus frequency is a straight line. The horizontal intercept is the critical frequency, f_c. **b** When another photo-surface is used, a line parallel to the first is obtained.

The final puzzling observation in these experiments is that the electrons are emitted immediately after the light is incident on the photo-surface, with no apparent time delay.

We now have four surprising observations:

1 The intensity of the incident light does not affect the energy of the emitted electrons.
2 The electron energy depends on the frequency of the incident light.
3 There is a certain minimum frequency below which no electrons are emitted.
4 Electrons are emitted with no time delay.

These four observations cannot be understood in terms of light as a wave for several reasons:

- If light is a wave, then an intense beam of light carries a lot of energy and so it should cause the emission of electrons that have more energy.

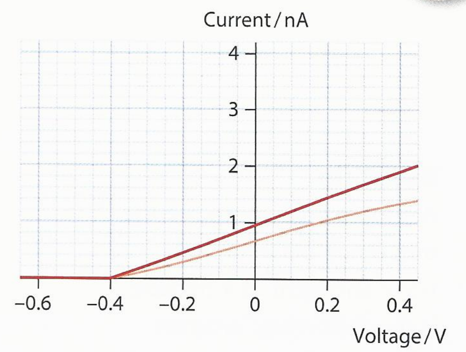


Figure 12.3 The stopping voltage for weak light (thin line) and intense light (thick line) of the same frequency.

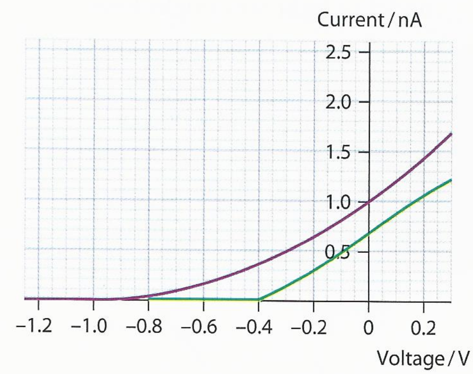


Figure 12.4 The stopping voltages for green and violet light.

Exam tip
A simple analogy to see the difference between light as a wave and light as a particle is the following: imagine winning a huge amount of money in a lottery, say 100 million euro. If this were to be given to you in the wave model of light you might have to wait a very long time to get all the money, if the money were paid to you at a rate of one million euro a year. In the photon model, all the money would be given to you at once.

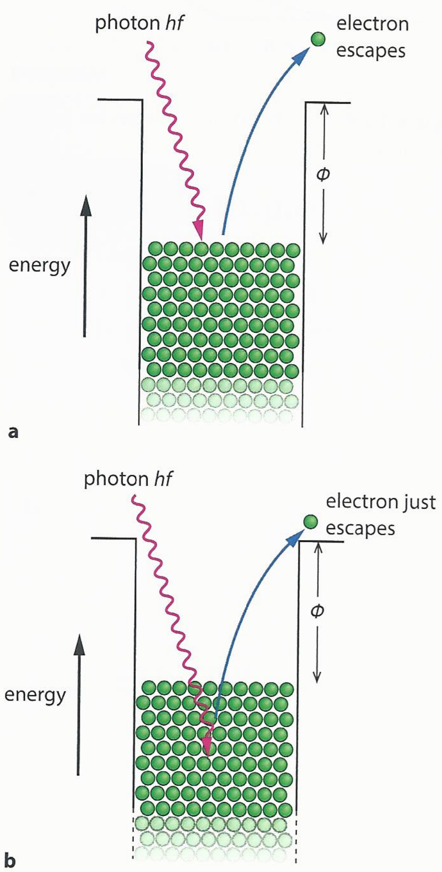


Figure 12.6 **a** A single photon of light may release a single electron from a metal. **b** A more tightly bound electron needs more energy to release it from the metal.

- The formula for the energy of a light wave does not include the frequency, and so frequency should play no role in the energy of the emitted electrons. In the same way there can be no explanation of a critical frequency.
- Finally, a very low intensity beam of light carries little energy. An electron might have to wait for a considerable length of time before it accumulated enough energy to escape from the metal. This would cause a delay in its emission.

Einstein's explanation

The explanation of all these strange observations was provided by Albert Einstein in 1905.

Einstein suggested that light consists of photons, which are **quanta or bundles of energy and momentum**. The energy of one such quantum is given by the formula:

$$E = hf$$

where f is the frequency of the electromagnetic radiation and $h = 6.63 \times 10^{-34}$ J s is a constant, known as Planck's constant.

Einstein's mechanism for the photoelectric effect is that a single photon of frequency f is absorbed by a single electron in the photo-surface, so the electron's energy increases by hf. The electron will have to spend a certain amount of energy, let us say Φ, to free itself from the pull of the nuclei of the atoms of the photo-surface. The electron will be emitted (become free) if hf is bigger than Φ. The difference $hf - \Phi$ will simply be the kinetic energy E_K of the (now) free electron (Figure **12.6**). That is:

$$E_K = hf - \Phi$$

The value of Φ (called the **work function**) is read off the graph, from the intercept of the straight line with the vertical axis. Note that the work function and the critical frequency are related by:

$$hf_c = \Phi$$

since $E_K = 0$ in that case.

In the photoelectric apparatus, the maximum kinetic energy of the electrons is measured to be $eV_s = E_{max}$. So:

$$E_{max} = hf - \Phi$$

It follows that:

$$eV_s = hf - \Phi$$

$$V_s = \frac{h}{e}f - \frac{\Phi}{e}$$

That is, in a graph of stopping voltage versus frequency, the graph is a straight line with slope h/e.

Worked examples

> **Exam tip**
> Remember to use energy in joules to calculate the critical frequency.

12.4 A photo-surface has a work function of 1.50 eV.

a Determine the critical frequency.

b Light of frequency 6.10×10^{14} Hz falls on this surface. Calculate the energy and speed of the emitted electrons.

a The critical frequency f_c is given in terms of the work function by $hf_c = \Phi$ and thus:

$$f_c = \frac{\Phi}{h} = \frac{1.50 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}}$$

$$f_c = 3.62 \times 10^{14}\,\text{Hz}$$

b The maximum kinetic energy of the electron is $E_{max} = hf - \Phi$, i.e.

$E_{max} = hf - hf_c = h(f - f_c)$

$$E_{max} = 6.63 \times 10^{-34} \times (6.10 - 3.62) \times 10^{14} = 1.64 \times 10^{-19}\,\text{J} \quad (= 1.03\,\text{eV})$$

From $E = \frac{1}{2}mv^2$ we find:

$$v = \sqrt{\frac{2E_{max}}{m}}$$

$$v = \sqrt{\frac{22 \times 1.64 \times 10^{-19}}{9.1 \times 10^{-31}}} \quad \text{(Use joules for } E_{max} \text{ to find } v.)$$

$$\Rightarrow \quad v = 6.0 \times 10^{5}\,\text{m s}^{-1}$$

12.5 Monochromatic light of power P and wavelength 4.0×10^{-7} m falling on a photo-surface whose critical frequency is 6.0×10^{14} Hz releases 2.0×10^{10} electrons per second.

a Determine the current collected in the anode.

b The power of the light is increased to $2P$. Predict the value of the new current.

c Light of power $2P$ and wavelength 6.0×10^{-7} m falls on this photo-surface. Determine the current in this case.

a The definition of electric current is $I = \frac{\Delta q}{\Delta t}$.

In a time of 1 second, the number of electrons emitted is 2.0×10^{10} and so the charge they carry is $e \times 2.0 \times 10^{10}$.

The current is thus $I = e \times 2.0 \times 10^{10}$, i.e.

$$I = 3.2 \times 10^{-9}\,\text{A}.$$

b If the power doubles, the number of photons will double and so the number of electrons emitted will double. Thus, so will the current, giving $I = 6.4 \times 10^{-9}$ A.

c The critical frequency f_c is 6.0×10^{14} Hz . From the wave equation, $c=f_c\times$ critical wavelength.

So the critical wavelength is:

$$\lambda_c=\frac{c}{f_c}=\frac{3\times10^{8}}{6.0\times10^{14}}=5.0\times10^{-7}\,\text{m}$$

So if the wavelength becomes 6.0×10^{-7} m, no electrons will be emitted at all, hence $I=0$.

12.6 The green light in Figure **12.4** has a wavelength of 496 nm.

a Determine the work function of the photo-surface.

b Estimate the wavelength of the violet light in that experiment.

a The stopping voltage is 0.40 V and so, using $eV_s=hf-\Phi$ we deduce that:

$$\Phi=\frac{hc}{\lambda}-eV_s$$

$$\Phi=\frac{1.24\times10^{-6}}{4.96\times10^{-7}}-0.40$$

$$\Phi=2.10\,\text{eV}$$

b We again use $eV_s=hf-\Phi$ to get that $\frac{hc}{\lambda}=eV_s+\Phi$.

The stopping voltage is 1.0 eV and so:

$$\frac{hc}{\lambda}=1.0+2.1$$

$$\frac{hc}{\lambda}=3.1\,\text{eV}$$

Hence:

$$\lambda=\frac{hc}{3.10}$$

$$\lambda=\frac{1.24\times10^{-6}}{3.1}$$

$$\lambda=4.0\times10^{-7}\,\text{m}$$

Exam tip

Notice the use of $hc=1.24\times10^{-6}$ eV m, which makes calculations much faster. This constant is in the IB data booklet.

Matter waves

In 1923, Louis de Broglie suggested that to any particle of momentum p, there corresponds a wave of wavelength given by the formula (h is Planck's constant):

$$\lambda=\frac{h}{p}$$

The de Broglie hypothesis, as this is known, thus assigns wave-like properties to something that is normally thought to be a particle. This state of affairs is called the **duality of matter**. All moving particles (not just electrons) are assigned a wavelength.

What does it mean to say that the electron has wave-like properties? One thing it does not mean is to think that the electron oscillates up and down as it moves along.

Showing wave-like properties means showing the basic phenomena of waves: diffraction and interference. A wave of wavelength λ will diffract around an obstacle of size d if λ is comparable to or bigger than d. In Worked example **12.7** we calculated a typical electron wavelength to be of order 10^{-10} m. This distance is typical of the separation of atoms in crystals, and it is there that electron diffraction and interference will be seen.

Worked example

12.7 Find the de Broglie wavelength of an electron that has been accelerated from rest by a potential difference of 54 V.

The kinetic energy of the electron is given by $E_K = \dfrac{p^2}{2m}$

The work done in accelerating the electron through a potential difference V is qV, and this work goes into kinetic energy. Thus:

$$\frac{p^2}{2m} = qV$$

$$\Rightarrow \quad p = \sqrt{2mqV}$$

Hence:

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

$$\lambda = \frac{6.63\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.60\times10^{-19}\times54}}$$

$$\lambda = 1.7\times10^{-10}\ \text{m}$$

Exam tip

It is preferable to use $E_K = \dfrac{p^2}{2m}$ for kinetic energy rather than $E_K = \frac{1}{2}mv^2$.

Exam tip

The formula $\lambda = \dfrac{h}{\sqrt{2mqV}}$ is very useful in paper 1 questions, where it is often required to know that $\lambda \propto \dfrac{1}{\sqrt{V}}$

Davisson and Germer investigated the scattering of low-energy electrons from a nickel surface. Initial results showed that, for fixed electron energy, the intensity of the electron beam decreased sharply as the scattering angle θ increased. A container of liquid air was accidentally dropped, breaking the glass jar housing the apparatus and exposing the nickel surface (which was surrounded by vacuum) to air, oxidising it. To remove the oxide, Davisson and Germer heated the surface in an atmosphere of hydrogen. The scattering of electrons was continued but now the results were very different. The intensity of the scattered electron beam varied strongly with scattering angle. After much thought, Davisson and Germer realised that they were dealing with scattering from a single crystal of nickel (that had grown on the surface as a result of heating it). Using crystals of known interatomic spacing, they eventually concluded they were seeing of electron diffraction with a wavelength given by the de Broglie formula.

Experiments showing the wave nature of the electron were carried out in 1927 by Clinton J. Davisson (1881–1958) and Lester H. Germer (1896–1971), and also by George Thomson (1892–1975), son of J.J. Thomson, the discoverer of the electron. In the Davisson–Germer experiment, electrons of kinetic energy 54 eV were directed at a surface of nickel where a single crystal had been grown and were scattered by it (Figure **12.7**).

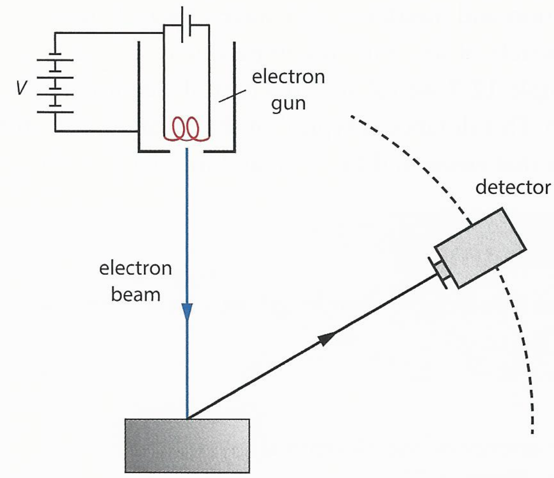


Figure 12.7 The apparatus of Davisson and Germer. Electrons emitted from the hot filament of the electron gun are accelerated through a known potential difference V and are then allowed to fall on a crystal. The positions of the scattered electrons are recorded by a detector.

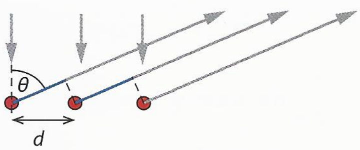


Figure 12.8 Electrons scattering off the top layer of atoms in a crystal will interfere. The path difference is shown in blue.

Because the electron energy is low, the electrons could not penetrate the crystal and were scattered by just the top layer of atoms (Figure **12.8**). The path difference between successive scattered electrons is $d\sin\theta$. When this is an integer multiple of the wavelength, we have constructive interference (the argument is the same as that given in Topic **9** for the diffraction grating):

$$d\sin\theta = n\lambda$$

In the Davisson–Germer experiment the distance d was known to be 0.215 nm. The first maximum ($n = 1$) was observed at an angle $\theta = 54°$. This allows determination of the wavelength:

$$\lambda = d\sin\theta = 0.215\times10^{-9}\times\sin 54° = 1.7\times10^{-10}\,\text{m}$$

We have already calculated the de Broglie wavelength of the electron that had been accelerated by a potential difference 54 V in Worked example **12.7**; it was found to be 1.7×10^{-10} m. This is in excellent agreement with the experiment, thus verifying the de Broglie hypothesis.

Pair annihilation and pair production

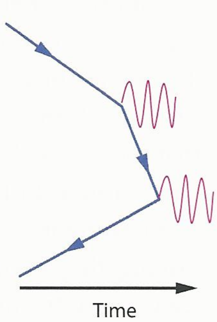


Figure 12.9 Feynman diagram for pair annihilation.

One of the striking features of quantum theory is the ability to convert matter into energy and vice versa. We know that for every particle there exists an anti-particle with the same mass (but opposite all other properties). What would happen if a particle collided with its anti-particle (Figure **12.9**)? This process is known as **pair annihilation**.

Consider for simplicity an electron of kinetic energy E_K that collides with a positron (the anti-particle of the electron) that moves in the opposite direction with the same kinetic energy. The total energy of the electron–positron system before they collide is $E_T = 2(mc^2 + E_K)$. This energy will be converted into the energy of two photons: the photons must be moving with the same energy in opposite directions and so they have the same wavelength:

$$\lambda = \frac{hc}{mc^2} + E_K$$

The longest wavelength will be emitted when the particles are more or less at rest, $E_K = 0$, and so in this case:

$$\lambda = \frac{hc}{mc^2}$$

$$\lambda = \frac{1.24 \times 10^{-6}}{0.511 \times 10^6}$$ (recall from the IB data booklet that, for the electron, $mc^2 = 0.511 \times 10^6$ eV)

$$\lambda = 2.4 \times 10^{-12}\,\text{m}$$

A **single** photon cannot materialise into a particle–anti-particle pair because such a process cannot conserve energy and momentum. But a single photon can make use of a nearby nucleus (Figure **12.10**) to produce a particle–anti-particle pair. The presence of the nucleus helps conserve energy and momentum. This process of **pair creation**, in effect, is a case where energy is converted into matter.

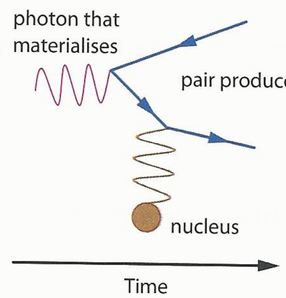


Figure 12.10 Feynman diagram for pair production. A nearby nucleus is required.

Worked example

12.8 **a** Estimate the wavelength of a photon that can just produce an electron–positron pair.
b Explain why this is only an estimate and not an accurate result.

a 'Just' producing the pair means producing it at rest. Thus the energy that needs to be provided is just the rest energy of particle, i.e. $2mc^2$. This energy is therefore $2 \times 0.511 = 1.02$ MeV.

The energy of a photon is $\frac{hc}{\lambda}$ and so:

$$\frac{hc}{\lambda} = 1.02 \times 10^6$$

$$\lambda = \frac{hc}{1.02 \times 10^6}$$

$$\lambda = \frac{1.24 \times 10^{-6}\,\text{eV m}}{1.02 \times 10^6\,\text{eV}}$$

$$\lambda = 1.2 \times 10^{-12}\,\text{m}$$

b This is only an estimate because one photon by itself cannot create the pair. It needs the presence of a nucleus that will share in energy and momentum conservation. The answer in **a** has not taken into account the nucleus.

Figure 12.11 A young Niels Bohr.

Exam tip

There is a lot of algebra in this derivation that must be learned carefully.

There are clear similarities here with the work done in Topic **10** on orbital motion in gravitation.

Quantisation of angular momentum

Niels Bohr (1885–1962) was a Danish physicist who studied the hydrogen atom (Figure **12.11**). This is the simplest atom, consisting of a nucleus of a single proton and a single electron orbiting it (Figure **12.12**).

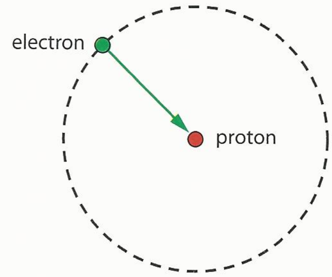


Figure 12.12 An electron orbiting a proton. The force on the electron is the electric force.

Let us calculate the total energy E_T of the orbiting electron. It is:

$$E_T = \underbrace{\frac{1}{2}mv^2}_{\text{kinetic}} + \underbrace{\left(-\frac{ke^2}{r}\right)}_{\text{electric potential}}$$

But the electron is acted upon by the electric force, and so:

$$\frac{ke^2}{r^2} = \frac{mv^2}{r}$$

From this we deduce that $mv^2 = \frac{ke^2}{r}$, and so the total energy becomes:

$$E_T = \frac{1}{2}\frac{ke^2}{r} - \frac{ke^2}{r}$$

$$E_T = -\frac{1}{2}\frac{ke^2}{r}$$

At this point Bohr made the revolutionary assumption that the **angular momentum** of the orbiting electron, i.e. the quantity $L = mvr$, is quantised. By this he meant that L is an integral multiple of a basic unit, the unit being $\frac{h}{2\pi}$. Here h is Planck's constant. The Bohr condition is therefore:

$$mvr = \frac{nh}{2\pi}$$

If we accept this for a moment, then we have that:

$$m^2v^2r^2 = \frac{n^2h^2}{4\pi^2}$$

and so $mv^2 = \frac{n^2h^2}{4\pi^2 mr^2}$

But earlier we found that $mv^2 = \frac{ke^2}{r}$, so substituting for mv^2 we get:

$$\frac{ke^2}{r} = \frac{n^2h^2}{4\pi^2mr^2}$$

This gives the extraordinary result that the orbital radius cannot be anything we wish: it equals

$$r = \frac{h^2}{4\pi^2ke^2m} \times n^2$$

Putting in the constants (using slightly more accurate values than those listed in the data booklet) gives:

$$r = \frac{(6.626 \times 10^{-34})^2}{4\pi^2 \times 8.988 \times 10^9 \times (1.602 \times 10^{-19})^2 \times 9.109 \times 10^{-31}} \times n^2$$

$$r = 0.5 \times 10^{-10} \times n^2\,\text{m}$$

What is now even more extraordinary is that the total energy of the orbiting electron is:

$$E = -\frac{2\pi^2me^4k^2}{h^2} \times \frac{1}{n^2}$$

We can combine all the constants in the first term in the expression as C, to give:

$$E = -\frac{C}{n^2}$$

Here k is the constant in Coulomb's law, m is the mass of the electron, e is the charge of the electron and h is Planck's constant. Numerically (again, using slightly more accurate values) C equals:

$$C = \frac{2\pi^2(9.109 \times 10^{-31})(1.602 \times 10^{-19})^4(8.988 \times 10^9)^2}{(6.626 \times 10^{-34})^2} \times n^2$$

$$C = 2.170 \times 10^{-18}\,\text{J}$$

$$C = 13.6\,\text{eV}$$

So that finally, we obtain:

$$E = -\frac{13.6}{n^2}\,\text{eV}$$

In other words, the theory predicts that the electron in the hydrogen atom has discrete or **quantised energy**. As we saw in Topic **7**, this explains the emission and absorption spectra of hydrogen.

Worked examples

12.9 In gravitation the period of revolution T of a planet in a circular orbit of radius R around the Sun obeys $T^2 \propto R^3$. Deduce the corresponding relation in the Bohr hydrogen model for an electron.

From $mvr = \frac{nh}{2\pi}$ we see that $v \propto \frac{n}{r}$. This means that $\frac{2\pi r}{T} \propto \frac{n}{r}$ But $v \propto n^2$ and so $T \propto wr^{3/2}$ as in gravitation!

12.10 Before Bohr, Johann Balmer (1825–1898) deduced experimentally that the photons emitted in transitions from a level n to the level $n = 2$ of hydrogen have wavelengths given by:

$$\lambda = \frac{Bn^2}{n^2 - 4}$$

where B is a constant. Justify this formula on the basis of the Bohr theory for hydrogen and find an expression for the constant B.

Balmer considered transitions from an energy level n down to the energy level 2. Let the difference in energy of the electron in level n and level $n = 2$ be ΔE. Then:

$$\Delta E = -\frac{C}{n^2} - \left(-\frac{C}{2^2}\right)$$

where $C = \frac{2\pi^2 me^4 k^2}{h^2}$ This energy ΔE is equal to the energy of the emitted photon, i.e. $\frac{hc}{\lambda}$. Thus:

$$\frac{hc}{\lambda} = \frac{C}{n^2} - \left(-\frac{C}{2^2}\right)$$

$$\frac{1}{\lambda} = \frac{C}{hc}\left(\frac{1}{4} - \frac{1}{n^2}\right)$$

$$\frac{1}{\lambda} = \frac{C}{hc} \times \left(\frac{n^2 - 4}{4n^2}\right)$$

This implies finally that $\lambda = \frac{4hc}{C} \times \frac{n^2}{n^2 - 4}$

This is precisely Balmer's formula with $B = \frac{4hc}{C}$

12.11 Show that the Bohr condition for the quantisation of angular momentum is equivalent to $2\pi r = n\lambda$, where λ is the de Broglie wavelength of the electron and r the radius of its orbit.

The Bohr condition is that: $mvr = \frac{nh}{2\pi}$

This can be re-written as: $2\pi r = \frac{nh}{mv}$

But according to de Broglie, $\frac{h}{mv} = \lambda$, and so we have the result.

The result of Worked example **12.11** shows that the allowed orbits in the Bohr model of hydrogen are those for which an integral number of electron wavelengths fit on the circumference of the orbit. Figure **12.13** shows the electron wave for $n = 6$. The circle in blue is the actual orbit. The solid red and the dotted red lines show the extremes of the electron wave. This is reminiscent of standing waves: the electron wave is a standing wave on the circumference. We know that standing waves do not transfer energy. This is a partial way to understand why the electrons do not radiate when in the allowed orbits.

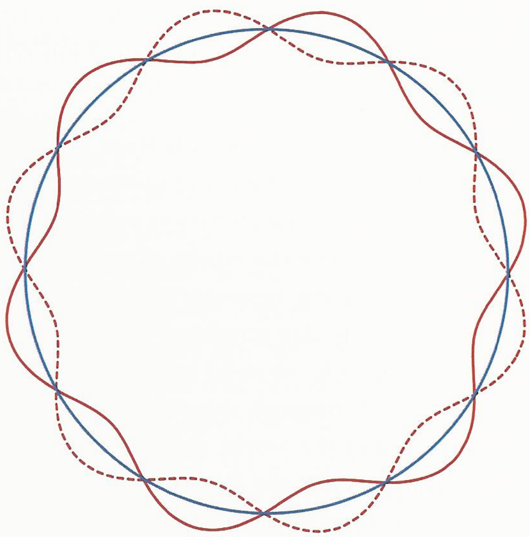


Figure 12.13 The allowed electron orbits are those for which an integral number of electron wavelengths fits on the circumference of the orbit.

The wavefunction

In the section on matter waves we said that particles exhibit wave-like behaviour; in the previous section we showed that the electron wave is a standing wave on the circumference of the orbit. But we have never specified what kind of waves we are talking about.

In 1926 the Austrian physicist Erwin Schrödinger (1887–1961) (Figure **12.14**) provided a realistic, quantum model for the behaviour of electrons in any atom – not just the hydrogen atom. The **Schrödinger theory** assumes as a basic principle that there is a wave associated with the electron (very much like de Broglie had assumed). This wave is called the **wavefunction**, $\psi(x, t)$, and is a function of position x and time t. Given the forces that act on the electron, it is possible, in principle, to solve a complicated differential equation obeyed by the wavefunction (the Schrödinger equation) and obtain $\psi(x, t)$. For example, there is one wavefunction for a free electron, another for an electron in the hydrogen atom, etc.

The interpretation of what $\psi(x, t)$ really means came from the German physicist Max Born (1882–1970). He suggested that the **probability** $P(x, t)$ that an electron will be found within a small volume ΔV near position x at time t is:

$$P(x, t) = |\psi(x, t)|^2 \Delta V$$

Figure 12.14 Erwin Schrödinger.

The theory only gives probabilities for finding an electron somewhere – it does not pinpoint an electron at a particular point in space. This is a radical change from classical physics, where objects have well-defined positions.

The Copenhagen interpretation of quantum mechanics

Through Bohr's own work and the numerous discussions of Bohr with his visitors at his institute at Blegdamsvej 17 in Copenhagen, the presently accepted interpretation of quantum mechanics is called the **Copenhagen interpretation**. It states that any physically meaningful quantity about a system can only be obtained from knowledge of its Schrödinger wavefunction, ψ. It also states that at any one time the system's wavefunction is a **superposition of all possible states** available to the system and that once a measurement is made that shows, for example, that the system has a particular momentum, then the wavefunction collapses to a wavefunction representing that particular momentum.

Not everyone has been comfortable with this interpretation. Schrödinger himself devised a situation that purports to show this interpretation is not sound. He thought of a cat in a box along with some radioactive atoms and a flask of poison. By some arrangement, if an atom decays the flask breaks releasing the poison and killing the cat. So the wavefunction of the cat is a superposition of the two states available to the cat, dead or alive. If we open the box and see that the cat is alive then the cat's wavefunction collapses to one representing a live cat. But before opening the box we don't know. This bothers many physicists. Physics Nobel prize winner Steven Weinberg says in a July 2013 interview in *Physics Today*:

> Some very good theorists seem to be happy with an interpretation of quantum mechanics in which the wavefunction only serves to allow us to calculate the results of measurements. But the measuring apparatus and the physicist are presumably also governed by quantum mechanics, so ultimately we need interpretive postulates that do not distinguish apparatus or physicists from the rest of the world, and from which the usual postulates like the Born rule can be deduced. This effort seems to lead to something like a 'many worlds' interpretation, which I find repellent. Alternatively, one can try to modify quantum mechanics so that the wavefunction does describe reality, and collapses stochastically and nonlinearly, but this seems to open up the possibility of instantaneous communication. I work on the interpretation of quantum mechanics from time to time, but have gotten nowhere.

So, finally, the kind of wave that we are referring to is a probability wave: a wave that gives the probability of finding a particle near a particular position. So when we say that the scattered electrons in the Davisson–Germer experiment interfere, what we mean is that the probability waves of the electrons interfere.

When the Schrödinger theory is applied to the electron in a hydrogen atom, it gives all the results that Bohr derived (the correct energy levels, for example). But it also predicts the probability that a particular transition will occur. This is necessary in order to understand why some spectral lines are brighter than others. Thus the Schrödinger theory explains atomic spectra for hydrogen and all other elements.

The uncertainty principle

The Heisenberg uncertainty principle is named after Werner Heisenberg (1901–1976), one of the founders of quantum mechanics (Figure **12.15**). He discovered the principle in 1927. The basic idea behind it is the wave–particle duality. Particles sometimes behave like waves and waves sometimes behave like particles, so that we cannot cleanly divide physical objects as either particles or waves.

Figure 12.15 Werner Heisenberg.

Duality

We have seen conflicting descriptions of physical objects. In Topic **9** we saw clear evidence that light behaves as a wave. In Topic **12** we see that light behaves as particles. In Topic **5** the motion of electrons in electric and magnetic fields was seen to obey the laws of Newtonian particle mechanics. In Topic **12** de Broglie tells us that electrons diffract the way waves do. This state of affairs is called the duality of matter – it shows the inadequacy of ordinary language to provide adequate descriptions of physical objects. It is made worse when we realise that two-slit interference experiments have been performed with light that is so weak that photons go through the slits one at a time. If so, what is the one photon going through a slit at a particular instant of time interfering with? Similar arguments may be made for electrons going through slits one at a time. The way out is to insist that the correct description during the passage through the slits is the wave description. In that case we can understand interference because a wave describes the object through the slits and the wave, because of its spread-out wavefront, covers both slits.

The Heisenberg uncertainty principle applied to position and momentum states that it is not possible to measure simultaneously the position and momentum of a particle with indefinite precision. This has nothing to do with imperfect measuring devices or experimental errors. It represents a fundamental property of nature. The uncertainty Δx in position and the uncertainty Δp in momentum are related by:

$$\Delta x \Delta p \geq \frac{h}{4\pi}$$

where h is Planck's constant.

This says that making momentum as accurate as possible makes position inaccurate, whereas accuracy in position results in inaccuracy in momentum. In particular, if one is made zero, the other has to be infinite.

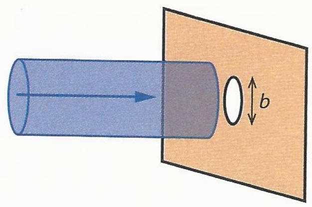


Figure 12.16 The narrower the beam, the smaller the uncertainty in the vertical position of an electron.

Where does a formula like this come from? To get a rough answer consider the following argument due to Heisenberg. Imagine a horizontal beam of electrons travelling towards a circular aperture (Figure **12.16**). We wish to make this beam as narrow as possible.

When the beam is made narrow the uncertainty in the vertical position of an electron is reduced. We can have a beam of width b if we let the beam go through a hole of diameter b. The uncertainty in position Δx is then:

$$\Delta x \approx \frac{b}{2}$$

We can make the electron beam as thin as possible by making the opening as small as possible.

However, we will run into a problem as soon as the opening becomes of the same order as the de Broglie wavelength of the electrons. A wave of wavelength λ will **diffract** when going through an aperture of about the same size as the wavelength. The electron will diffract through the opening, which means that a few electrons will emerge from the opening with a direction that is no longer horizontal.

We can describe this phenomenon by saying that there is an uncertainty in the electron's momentum in the vertical direction, of magnitude Δp. Figure **12.17** shows that there is a spreading of the electrons within an angular size 2θ.

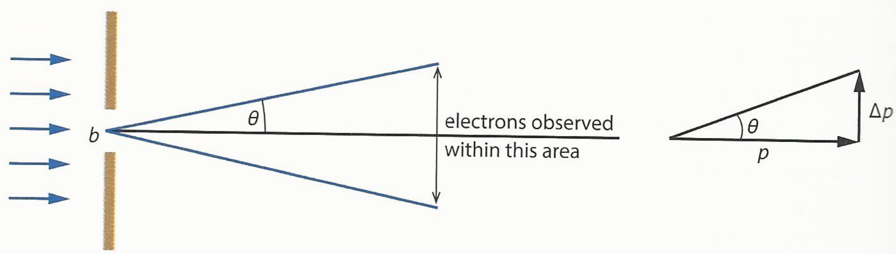


Figure 12.17 An electron passing through a slit suffers a deflection in the vertical direction.

The angle by which the electron is diffracted is given by:

$$\theta \approx \frac{\lambda}{b}$$

where b is the opening size. But from Figure **12.17**, $\theta \approx \frac{\Delta p}{p}$. Therefore:

$$\frac{\lambda}{b} \approx \frac{\Delta p}{p}$$

But $b \approx 2\Delta x$ so:

$$\frac{\lambda}{2\Delta x} \approx \frac{\Delta p}{p}$$

$$\Rightarrow \quad \Delta x \Delta p \approx \frac{\lambda p}{2}$$

The de Broglie wavelength is given by $\lambda = \frac{h}{p}$. So:

$$\Delta x \Delta p \approx \frac{h}{2}$$

This is a simple explanation of where the uncertainty formula comes from. (This is derivation is only approximate, which is why we are missing a factor of 2π.)

Worked example

12.12 A very fine beam of electrons with speed $10^6\,\text{m s}^{-1}$ are directed horizontally towards a slit whose opening is 10^{-10} m. Electrons are observed on a screen at distance of 1 m from the slit. Estimate the length on the screen where appreciable numbers of electrons will be observed.

There is an uncertainty of 10^{-10} m in the vertical component of the position of the electron. Therefore there will be an uncertainty in the vertical component of momentum of:

$$\Delta p_y \approx \frac{6.63\times10^{-34}}{4\pi\times10^{-10}} \approx 5\times10^{-25}\,\text{N s}$$

The momentum of the electrons is $p \approx 9.1\times10^{-31}\times10^{6} \approx 9\times10^{-25}$ N s. The electrons will therefore be deviated by an angle θ given by:

$$\theta \approx \frac{\Delta p_y}{p} \approx \frac{5\times10^{-25}}{9\times10^{-25}} \approx 0.5\,\text{rad}$$

The electrons will therefore be observed in region of length $2\times0.5\times1=1$ m.

'Electron in a box'

As an application of the uncertainty principle, consider an electron confined in a region of size L. The electron can only move back and forth along a straight line of length L. Then the uncertainty in position must satisfy $\Delta x \approx \frac{L}{2}$, and so the uncertainty in momentum must be:

$$\Delta p \approx \frac{h}{4\pi\Delta x} \approx \frac{h}{2\pi L}$$

The electron must then have a kinetic energy of:

$$E_K = \frac{p^2}{2m} \approx \frac{\Delta p^2}{2m} = \frac{h^2}{8\pi^2 m L^2}$$

We may apply this result to an electron in the hydrogen atom. The size of the region within which the electron is confined is about $L \approx 10^{-10}$ m. Then:

$$E_K \approx \frac{h^2}{8\pi^2 m L^2} = \frac{(6.6\times10^{-34})^2}{8\pi^2(9.1\times10^{-31})(10^{-10})^2}$$

$$E_K \approx 6\times10^{-19}\,\text{J} \approx 4\,\text{eV}$$

which is in fact just about right for the electron's kinetic energy. This shows that the uncertainty principle is an excellent tool for making approximate estimates for various quantities.

Exam tip

An uncertainty in position Δx implies an uncertainty in the momentum: $\Delta p \approx \frac{h}{4\pi\Delta x}$

Now the momentum will be measured to be $p_0 + \Delta p$. The least magnitude of p_0 is 0, and so the least possible magnitude of the momentum of the electron is Δp. The energy of the electron is then at least:

$$E_K \approx \frac{p^2}{2m} \approx \frac{\Delta p^2}{2m}$$

Exam tip

In the examination, the uncertainty relations will be used for rough estimates. In that case rough equalities rather than inequalities will do:

$$\Delta x \Delta p \approx \frac{h}{4\pi}$$

$$\Delta E \Delta t \approx \frac{h}{4\pi}$$

Uncertainty in energy and time

The uncertainty principle also applies to measurements of energy and time. If the energy of a state is measured with an uncertainty ΔE, then the lifetime of the state is of order Δt such that:

$$\Delta E \Delta t \geq \frac{h}{4\pi}$$

This is also applies to decaying particles, where ΔE is the uncertainty in the measured value of the energy released and Δt is the lifetime of the particle.

Worked example

12.13 In the decay $\rho^0 \rightarrow \pi^+ + \pi^-$ the uncertainty in the energy released is 153 MeV. Calculate the expected lifetime of the ρ^0 meson and hence identify the interaction through which the decay takes place.

We will apply the uncertainty relation for energy and time $\Delta t \approx \frac{h}{4\pi \Delta E}$ to get:

$$\Delta t \approx \frac{6.63 \times 10^{-34}}{4\pi \times 154 \times 10^6 \times 1.6 \times 10^{-19}}$$

$$\Delta t \approx 2 \times 10^{-24}\,\mathrm{s}$$

Lifetimes that are this short are typical of the strong interaction.

Figure 12.18 The total energy of the ball is not enough to go over the barrier so the ball will not go over the barrier.

Tunnelling

Consider a ball of mass 2 kg that moves with speed $10\,\mathrm{m\,s^{-1}}$ along the path shown in Figure **12.18**. The kinetic energy of the ball is 100 J. Ahead of the ball is a hill of height 6 m. If the ball was placed at the top of the hill it would have potential energy $mgh = 120\,\mathrm{J}$. The total energy of the ball is only 100 J and so we do not expect the ball to get to the top of the hill and roll down the other side.

The probability of finding the ball to the right of the hill is zero. The ball will be 'reflected' by the hill. The hill acts as a 'potential barrier' to the ball. It does not allow the ball to go over the barrier if the ball does not have enough energy to get to the top of the barrier.

In microscopic physics the corresponding situation might involve protons of total energy E that face a region of positive electric potential as shown in Figure **12.19**. If the electric potential is V then the energy needed by one proton to go over the barrier is eV.

We expect that if the total energy of the proton is less than eV the proton has zero probability of moving from region A, through region B and into the 'forbidden' region C.

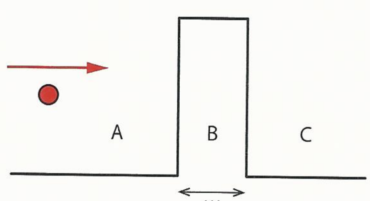


Figure 12.19 A potential barrier to a proton. The energy needed to go over is eV.

But one of the most impressive phenomena of quantum mechanics, **tunnelling**, makes this possible. This is intimately related to the fact that particles have wave properties and are described by wavefunctions. The Schrödinger theory must be used to determine the wavefunction of the protons in each of the three regions A, B and C in such a way that the wavefunctions in the three regions join smoothly (no jumps and no corners). This makes it necessary to have a non-zero wavefunction in region C. It is as if the wavefunction 'leaks' into region C. The probabilities are shown in Figure **12.20**.

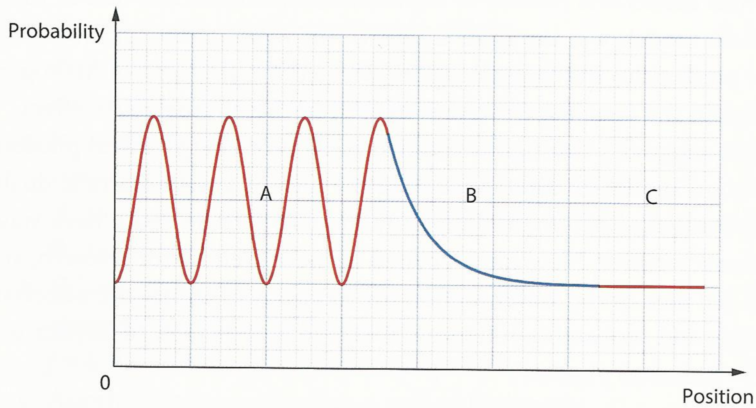


Figure 12.20 The wavefunction 'leaks' into the forbidden region C. (The graph shows the probability for finding the electron, not the wavefunction itself.)

In region A we see an oscillating probability. This is evidence for the presence of a standing wave: the wavefunction of the incoming protons gets superposed with that of the reflected protons. In region B the probability is exponentially decreasing (shown in blue). At the end of region B and the beginning of region C the probability is not zero. There is a non-zero wavefunction that describes the transmitted protons, those that tunnelled through the potential barrier. There is a small but non-zero probability of finding protons in the forbidden region C.

Three factors affect the probability of transmission:

1 the mass m of the particles
2 the width w of the barrier
3 the difference ΔE between the energy of the barrier and that of the particles.

The larger each of these quantities is, the smaller the transmission probability. In fact it is known that $p \propto \exp(-w\sqrt{m\Delta E})$. So everything else being equal, the transmission probability for electrons is greater than for protons, for example.

It is important to realise that the particles that emerge in region C have the same energy as they did in region A! Thus the de Broglie wavelength in region C is the same as in region A. Strange as it seems, the tunnelling phenomenon has very many practical applications, including the scanning tunnelling microscope (the microscope that can 'see' atoms) (Figure **12.21**) and the tunnel diode (a diode in which the current can be very quickly switched between on and off).

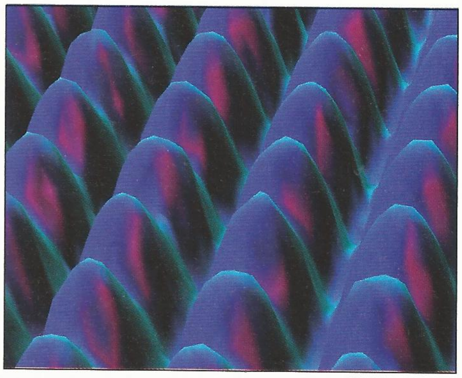


Figure 12.21 Scanning tunnelling microscope image of nickel atoms. (Image originally created by IBM Corporation.)

Nature of science

Quantum physics

The study of emission spectra from flames revealed characteristic lines that could be used to identify different elements. Observations of the spectrum of sunlight showed dark absorption lines. In the 1800s improved instruments allowed for more accurate measurements of the wavelengths of light corresponding to the bright and dark lines in spectra. But what was the reason for these lines? As we saw in Topic **7**, a quantum model was needed to explain these patterns. The first of these quantum models was the Bohr model of the hydrogen atom, which could predict the wavelengths of the lines in the spectrum of hydrogen. To explain the puzzling observations seen in the photoelectric effect, Einstein suggested that light existed as packets of energy called photons – a quantum model for light. Accepting the idea of a wave–particle duality revolutionised scientific thinking. Any moving particle could have wave-like characteristics! New ideas opened up new avenues for research, and led to the idea of the wavefunction for electrons in atoms, the uncertainty principle and probability functions. A whole new branch of physics was born – quantum physics.

? Test yourself

1 **a** Explain what is meant by the **photoelectric effect**.

b A photo-surface has a work function of 3.00 eV. Determine the critical frequency.

2 **a** What evidence is there for the existence of photons?

b A photo-surface has a critical frequency of 2.25×10^{14} Hz. Radiation of frequency 3.87×10^{14} Hz falls on this surface. Deduce the voltage required to stop electrons from being emitted.

3 Light of wavelength 5.4×10^{7} m falls on a photo-surface and causes the emission of electrons of maximum kinetic energy 2.1 eV at a rate of 10^{15} per second. The light is emitted by a 60 W light bulb.

a Explain how light causes the emission of electrons.

b Calculate the electric current that leaves the photo-surface.

c Determine the work function of the surface.

d Estimate the maximum kinetic energy of the electrons when the intensity of the light becomes 120 W.

e Estimate the current from the photo-surface when the intensity is 120 W.

4 **a** State **three** aspects of the photoelectric effect that cannot be explained by the wave theory of light. For each, outline how the photon theory provides an explanation.

b Light of wavelength 2.08×10^{-7} m falls on a photo-surface. The stopping voltage is 1.40 V.

i Outline what is meant by **stopping voltage**.

ii Calculate the largest wavelength of light that will result in emission of electrons from this photo-surface.

5 **a** The intensity of the light incident on a photo-surface is doubled while the wavelength of light stays the same. For the emitted electrons, discuss the effect of this, if any, on **i** the energy and **ii** the number.

b To determine the work function of a given photo-surface, light of wavelength 2.3×10^{-7} m is directed at the surface and the stopping voltage, V_s, recorded. When light of wavelength 1.8×10^{-7} m is used, the stopping voltage is twice as large as the previous one. Determine the work function.

6 Light falling on a metallic surface of work function 3.0 eV gives energy to the surface at a rate of 5.0×10^{-4} W per square metre of the metal's surface. Assume that an electron on the metal surface can absorb energy from an area of about $1.0 \times 10^{-18}\,m^2$.

a Estimate how long will it will take the electron to absorb an amount of energy equal to the work function.

b Outline the implication of this.

c Describe how the photon theory of light explains the fact that electrons are emitted almost instantaneously with the incoming photons.

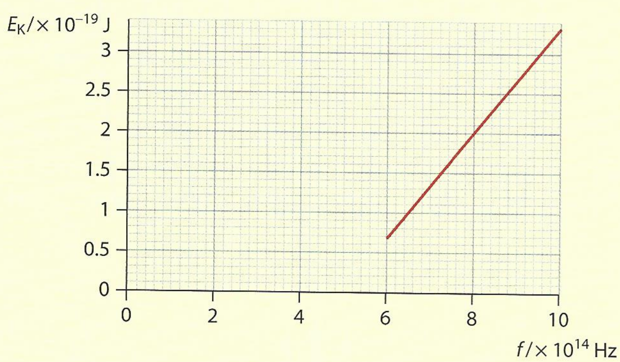
7 a From the graph of electron kinetic energy E_K versus frequency of incoming radiation, deduce:

i the critical frequency of the photo-surface

ii the work function.

b What is the kinetic energy of an electron ejected when light of frequency $f = 8.0 \times 10^{14}$ Hz falls on the surface?

c Another photo-surface has a critical frequency of 6.0×10^{14} Hz. On a copy of the graph below, sketch the variation with frequency of the emitted electrons' kinetic energy.



8 An electron of kinetic energy 11.5 eV collides with a hydrogen atom in its ground state. With what possible kinetic energy can this electron rebound off the atom?

9 This question will look at the intensity of radiation in a bit more detail. The intensity of light, I, incident normally on an area A is defined to be $I = \frac{P}{A}$, where P is the power carried by the light.

a Show that $I = \Phi hf$, where Φ is the photon flux density, i.e. the number of photons incident on the surface per second per unit area, and f is the frequency of the light.

b Calculate the intensity of light of wavelength $\lambda = 5.0 \times 10^{-7}$ m incident on a surface when the photon flux density is $\Phi = 3.8 \times 10^{18}\,m^{-2}\,s^{-1}$.

c The wavelength of the light is decreased to $\lambda = 4.0 \times 10^{-7}$ m. Calculate the new photon flux density so that the intensity of light incident on the surface is the same as that found in **b**.

d Hence explain why light of wavelength $\lambda = 4.0 \times 10^{-7}$ m and of the same intensity as that of light of wavelength $\lambda = 5.0 \times 10^{-7}$ m will result in fewer electrons being emitted from the surface per second.

e State **one** assumption made in reaching this conclusion.

10 a What is the evidence for the existence of energy levels in atoms?

b Electrons of kinetic energy **i** 10.10 eV, **ii** 12.80 eV and **iii** 13.25 eV collide with hydrogen atoms and can excite these to higher states. In each case, find the largest n corresponding to the state the atom can be excited to. Assume that the hydrogen atoms are in their ground state initially.

11 a What do you understand by the term **ionisation energy**?

b What is the ionisation energy for a hydrogen atom in the state $n = 3$?

12 a Find the smallest wavelength that can be emitted in a transition in atomic hydrogen.

b What is the minimum speed an electron must have so that it can ionise an atom of hydrogen in its ground state?

13 Consider a brick of mass 0.250 kg moving at $10\,m\,s^{-1}$.

a Estimate its de Broglie wavelength.

b Comment on whether it makes sense to treat the brick as a wave.

14 a Describe an experiment in which the de Broglie wavelength of an electron can be measured directly.

b Determine the speed of an electron whose de Broglie wavelength is equal to that of red light (680 nm).

15 **a** Show that the de Broglie wavelength of an electron that has been accelerated from rest through a potential difference V is given by:

$$\lambda = \frac{h}{\sqrt{2meV}}$$

b Calculate the ratio of the de Broglie wavelength of a proton to that of an alpha particle when both have been accelerated from rest by the same potential difference.

c Calculate the de Broglie wavelength of an electron accelerated from rest through a potential difference of 520 V.

16 **a** Find the de Broglie wavelength of a proton (mass 1.67×10^{-27} kg) whose kinetic energy is 200.0 MeV.

b What is the de Broglie wavelength of an electron in the $n = 2$ state of hydrogen?

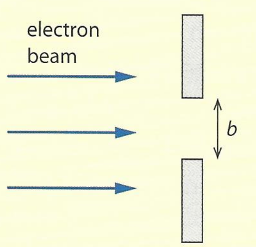
17 Using the uncertainty principle, show that an electron in a hydrogen atom will have a kinetic energy of a few eV.

18 **a** State the de Broglie hypothesis.

b Calculate the de Broglie wavelength of an electron that has been accelerated by a potential difference of 5.0 V.

c Explain why precise knowledge of the wavelength of an electron implies imprecise knowledge of its position.

19 An experimenter wishes to make a very narrow beam of electrons. To do that, she suggests the arrangement shown in the diagram. She expects that the beam can be made as narrow as possible by reducing the size b of the aperture through which the electrons will pass.



a Explain why in principle it is not possible to make a perfectly narrow beam.

b Are her chances of producing a narrow beam better with slow or fast electrons?

20 A tennis ball is struck so that it moves with momentum 6.0 N s straight through an open square window of side 1.0 m. Because of the uncertainty principle, the tennis ball may deviate from its original path after going through the window. Estimate the angle of deviation of the path of the tennis ball. Comment on your answer.

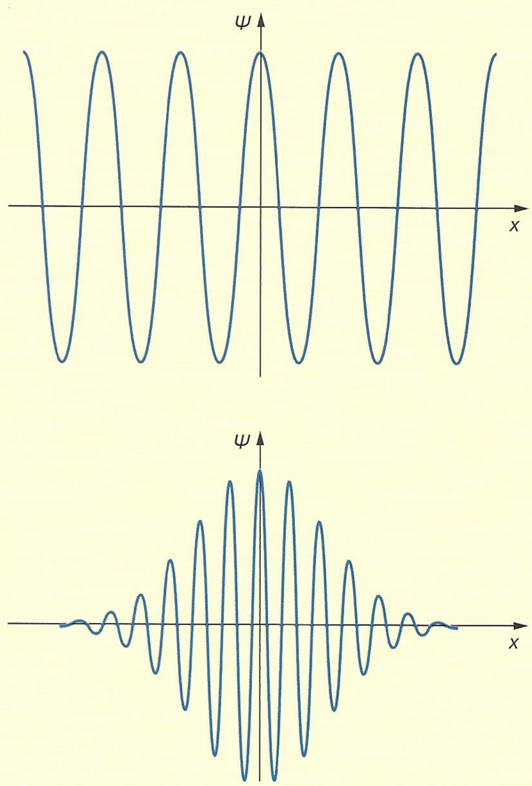
21 Theoretically it is possible in principle to balance a pencil on its tip so that it stands vertically on a horizontal table. Explain why in quantum theory this is impossible in principle. (You can turn this problem into a good theoretical extended essay if you try to estimate the time the pencil will stay up after it has been momentarily balanced!)

22 The graphs represent the wavefunctions of two electrons. Identify the electron with:

a the least uncertainty in momentum

b the least uncertainty in position.

Explain your answers.



23 Assume that an electron can exist within a nucleus (size 10^{-15} m) such that its associated wave forms a fundamental mode standing wave with nodes at the edges of the nucleus.

a Estimate the wavelength of this electron.

b Calculate the kinetic energy of the electron in MeV.

c Using your answer in **b**, comment on whether the electron emitted in beta-minus decay could have existed within the nucleus before the decay.