12.2 Nuclear physics

In this section we will examine ideas we met earlier in some more detail. These include Rutherford scattering (and more importantly, deviations from it), nuclear energy levels, the neutrino and the mathematics of radioactive decay.

Learning objectives

- Understand how the Rutherford scattering experiment led to the idea of the nucleus.
- Discuss how scattering experiments may be used to determine nuclear radii.
- Understand the nature of nuclear energy levels.
- Understand the nature of the neutrino.
- Solve problems involving the radioactive decay law.

Rutherford scattering

In scattering experiments such as Rutherford's (see Section **7.3**), simple energy considerations can be used to calculate the **distance of closest approach** of the incoming particle to the target. Consider, as an example, an alpha particle (of charge $q=2e$) that is projected head-on toward a stationary nucleus of charge $Q=Ze$ (Figure **12.22**).

Initially the system has a total energy consisting of the alpha particle's kinetic energy $E=E_K$. We take the separation of the alpha particle and the nucleus to be large so no potential energy exists. At the point of closest approach, a distance d from the centre of the nucleus, the alpha particle stops and is about to turn back. Thus, the total energy now is the electric potential energy of the alpha and the nucleus, given by:

$$E=k\frac{Qq}{d}$$

$$E=k\frac{(2e)(Ze)}{d}$$

$$E=k\frac{2Ze^2}{d}$$

(We are assuming that the nucleus does not recoil, so its kinetic energy is ignored.)

Then, by conservation of energy:

$$E_K=k\frac{2Ze^2}{d}$$

$$\Rightarrow \quad d=k\frac{2Ze^2}{E_K}$$

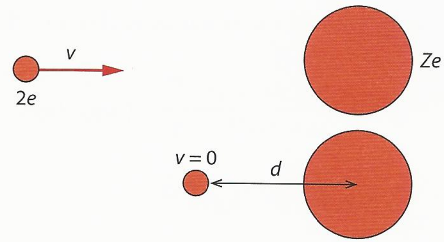


Figure 12.22 The closest approach of an alpha particle happens in a head-on collision.

Assuming a kinetic energy for the alpha particle equal to 2.0 MeV directed at a gold nucleus ($Z = 79$) gives $d = 1.1 \times 10^{-13}$ m. This is outside the range of the nuclear force, so the alpha particle is simply repelled by the electrical force.

If the energy of the incoming particle is increased, the distance of closest approach decreases. The smallest it can get will be the radius of the nucleus. Experiments of this kind have been used to estimate the nuclear radii. A result of these experiments is that the nuclear radius R depends on mass number A through:

$$R = R_0 A^{\frac{1}{3}} \quad \text{where} \quad R_0 = 1.2 \times 10^{-15}\,\text{m}$$

This has the unexpected consequence that all nuclei have the same density. Worked example **12.14** shows how this is derived.

Worked example

12.14 Show that all nuclei have the same density.

The volume is:

$$V = \frac{4\pi}{3} R^3$$

But $R = R_0 A^{\frac{1}{3}}$, so:

$$V = \frac{4\pi}{3}(1.2 \times 10^{-15} \times A^{\frac{1}{3}})^3$$

$$V = 7.24 \times 10^{-45} \times A\,\text{m}^3$$

The mass of the nucleus is A u, i.e. $A \times 1.66 \times 10^{-27}$ kg. Using density $= \frac{\text{mass}}{\text{volume}}$, the density is:

$$\rho = \frac{\cancel{A} \times 1.66 \times 10^{-27}}{7.24 \times 10^{-45} \times \cancel{A}} \quad \text{(note how } A \text{ cancels out)}$$

$$\rho \approx 2.3 \times 10^{17}\,\text{kg}\,\text{m}^{-3}$$

So all nuclei have the same density.

Another set of experiments aimed at determining nuclear radii involve sending beams of neutrons or electrons at nuclei. We know from diffraction that if the de Broglie wavelength λ of the electrons or neutrons is about the same as that of the nuclear diameter, the electrons and neutrons will diffract around the nuclei. A minimum will be formed at an angle θ to the original direction according to:

$$\sin\theta \approx \frac{\lambda}{b}$$

where b is the diameter of the diffracting object, i.e. the nucleus. (We met this relationship for diffraction in Topic **9**.) The advantage of using electrons is that the strong force does not act upon them and so they probe the nuclear charge distribution. Neutrons also have an advantage because, being neutral, they can penetrate deep into matter and get very close to the nucleus.

Worked example

12.15 In a neutron diffraction experiment, a beam of neutrons of energy 85 MeV are incident on a foil made out of lead and diffracted. The first diffraction minimum is observed at an angle of 13° relative to the central position where most of the neutrons are observed. From this information, estimate the radius of the lead nucleus.

The neutrons are diffracted from the lead nuclei, which act as 'obstacles' of size b. From our knowledge of diffraction, the first minimum is given by $\sin\theta \approx \frac{\lambda}{b}$, where λ is the de Broglie wavelength of the neutron.

The mass of a neutron is $m = 1.67 \times 10^{-27}$ kg and, since its kinetic energy is 85 MeV, the wavelength is $\lambda = \frac{h}{p}$ where:

$$p = \sqrt{2E_K m}$$

$$p = \sqrt{2 \times 85 \times 10^6 \times 1.6 \times 10^{-19} \times 1.67 \times 10^{-27}}$$

$$p = 2.13 \times 10^{-19}\,\text{N s}$$

Using this value of p in the equation for wavelength:

$$\lambda = \frac{6.6 \times 10^{-34}}{2.13 \times 10^{-19}}$$

$$\lambda = 3.1 \times 10^{-15}\,\text{m}$$

Therefore the diameter of the nucleus is given by:

$$b = \frac{\lambda}{\sin 13°} = \frac{3.1 \times 10^{-15}}{\sin 13°}$$

$$b = 14 \times 10^{-15}\,\text{m}$$

This corresponds to a radius of 7×10^{-15} m.

Deviations from Rutherford scattering

Rutherford derived a theoretical formula for the scattering of alpha particles from nuclei. The Rutherford formula states that as the scattering angle θ increases, the number of alpha particles scattered at that angle decreases very sharply. This is shown in Figure **12.23a**. In fact, Rutherford's formula states that the number N of alpha particles scattering at an angle θ is proportional to $1/\sin^4\left(\frac{\theta}{2}\right)$. If this is the case, the product $N\sin^4\left(\frac{\theta}{2}\right)$ should be constant. Table **12.1** contains some of the original

Angle of scattering, θ/°	N	$N\sin^4\left(\frac{\theta}{2}\right)$
15	132 000	38.4
30	7 800	35.0
45	1 435	30.8
60	477	29.8
75	211	29.1
105	69	27.5
120	52	29.0
150	33	28.8

Table 12.1 Data from the Geiger–Marsden experiment, reproduced in the book by E. Rutherford, J. Chadwick and C. D. Ellis *Radiations from Radioactive Substances*, Cambridge University Press, 1930.

data in the Geiger–Marsden experiment with a gold foil. The last column in the table shows that the product $N\sin^4\left(\frac{\theta}{2}\right)$ is indeed fairly constant, which is strong evidence in support of the Rutherford formula.

The derivation of the Rutherford formula is based on a number of assumptions. The most important is that the only force in play during the scattering process is the electric force. As the energy of the alpha particles increases, the alpha particles can get closer to the nucleus. When the distance of closest approach gets to be about 10^{-15} m or less, deviations from the Rutherford formula are observed (Figure **12.23b**). This is due to the fact that the alpha particles are so close to the nucleus that the strong nuclear force begins to act on the alpha particles.

Therefore, the presence of these deviations from perfect Rutherford scattering is evidence for the existence of the strong nuclear force.

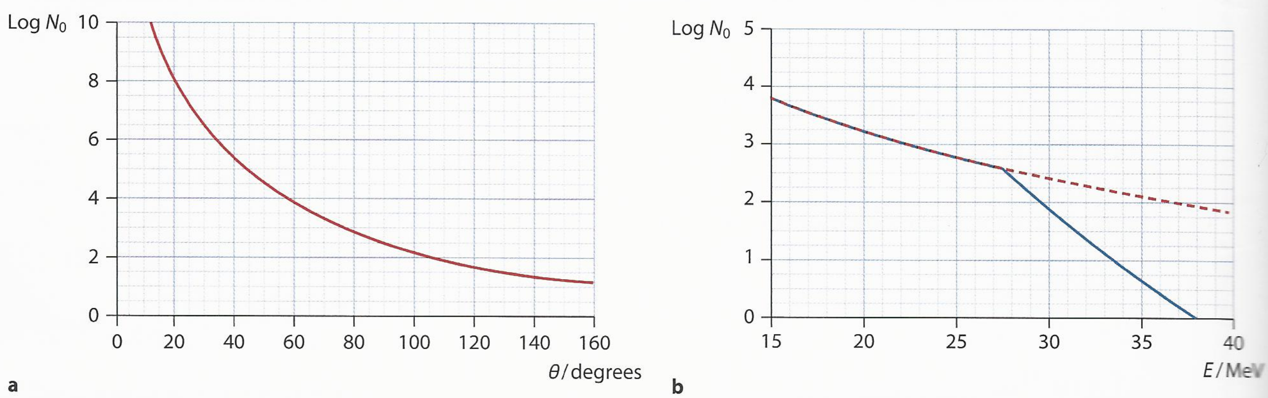


Figure 12.23 **a** The logarithm of the number of alpha particles scattered at some angle θ as a function of θ. **b** The logarithm of the number of alpha particles scattered at an angle of 60° as a function of the alpha particle energy. The dotted curve is based on Rutherford scattering. The blue curve is the observed curve. We see deviations when the energy exceeds about 28 MeV. The energy at which deviations start may be used to estimate the nuclear radius.

Worked example

12.16 Suggest how the results of the scattering of alpha particles would change if the gold ($^{197}_{79}Au$) foil was replaced by an aluminium ($^{30}_{13}Al$) foil of the same thickness.

Aluminium has a smaller nuclear charge and so the alpha particles would approach closer to the nucleus. This means that the alpha particles would start feeling the effects of the nuclear force and deviations from perfect Rutherford scattering would be observed.

Nuclear energy levels

The nucleus, like the atom, exists in discrete energy levels. The main evidence for the existence of nuclear energy levels comes from the fact that the energies of the alpha particles and gamma ray photons that are emitted by nuclei in alpha and gamma decays are **discrete**. (This is to be contrasted with beta decays, in which the electron has a continuous range of energies.)

Figure **12.24** shows the lowest nuclear energy levels of the magnesium nucleus $^{24}_{12}\text{Mg}$. Also shown is a gamma decay from the level with energy 5.24 MeV to the first excited state. The emitted photon has energy $5.24 - 1.37 = 3.87$ MeV.

Figure **12.25** shows an energy level of plutonium ($^{242}_{94}\text{Pu}$) and a few of the energy levels of uranium ($^{238}_{92}\text{U}$). Also shown are two transitions from plutonium to uranium energy levels. These are alpha decays:

$$^{242}_{94}\text{Pu} \rightarrow {}^{238}_{92}\text{U} + {}^{4}_{2}\alpha$$

The energies of the emitted alpha particles are $4.983 - 0.148 = 4.835$ MeV and $4.983 - 0.307 = 4.676$ MeV.

Worked example **12.17**, overleaf applies these ideas to beta decay.

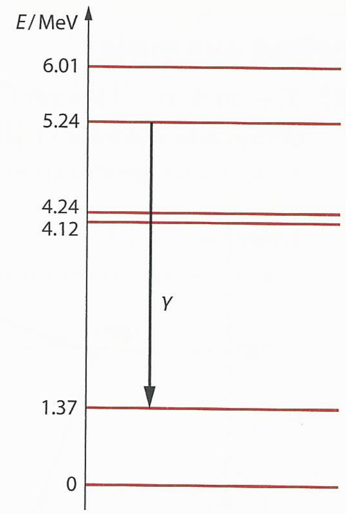


Figure 12.24 Nuclear energy levels of magnesium, $^{24}_{12}\text{Mg}$. Notice the difference in scale between these levels and atomic energy levels.

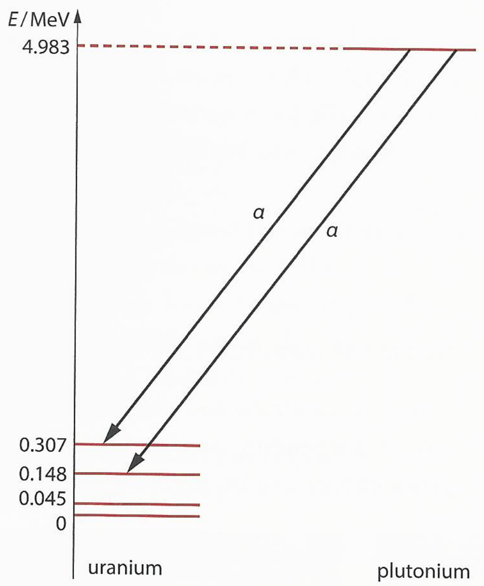


Figure 12.25 Energy levels for plutonium and uranium. Transitions from plutonium to uranium energy levels explain the discrete nature of the emitted alpha particle in the alpha decay of plutonium.

Worked example

12.17 The nucleus of bismuth ($^{211}_{83}\text{Bi}$) decays into lead ($^{207}_{82}\text{Pb}$) in a two-stage process. In the first stage, bismuth decays into polonium ($^{211}_{84}\text{Po}$). Polonium then decays into lead. The nuclear energy levels that are involved in these decays are shown in Figure **12.26**.

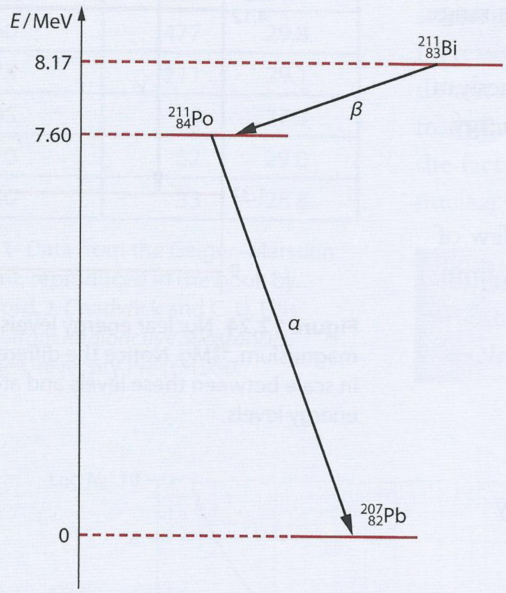


Figure 12.26 The two-stage decay of bismuth into lead.

a Write down the reaction equations for each decay.
b Calculate the energy released in the beta decay.
c Explain why the electron does not always have this energy.

a $^{211}_{83}\text{Bi} \rightarrow {}^{211}_{84}\text{Po} + {}^{0}_{-1}\text{e} + {}^{0}_{0}\bar{\nu}$ and $^{211}_{84}\text{Po} \rightarrow {}^{207}_{82}\text{Pb} + {}^{4}_{2}\alpha$

b The energy released is the difference in the energy levels involved in the transition, i.e. 0.57 MeV.

c The energy of 0.57 MeV must be shared between the electron, the anti-neutrino and the polonium nucleus. So the electron does not always have the maximum energy of 0.57 MeV. Depending on the angles (between the electron, the anti-neutrino and the polonium nucleus), the electron energy can be anything from zero up to the maximum value found in **b**.

The neutrino

In the 1930s it was thought that beta minus decay was described by:

$$^{1}_{0}\text{n} \rightarrow {}^{1}_{1}\text{p} + {}^{0}_{-1}\text{e}$$

The mass difference for this decay is:

$$1.008665\,\text{u} - (1.007276 + 0.0005486)\,\text{u} = 0.00084\,\text{u}$$

and corresponds to an energy of:

$$0.00084 \times 931.5\,\text{MeV} = 0.783\,\text{MeV}$$

If only the electron and the proton are produced, then the electron, being the lighter of the two, will carry most of this energy away as kinetic energy. To see this, assume that the neutron is at rest when it decays. Then the total momentum before the decay is zero. After the decay the electron and the proton will have equal and opposite momenta, each of magnitude p. Equating the kinetic energy after the decay to the energy released, Q:

$$\frac{p^2}{2m_e} + \frac{p^2}{2m_p} = Q$$

$$p^2 = \frac{2Qm_e m_p}{m_e + m_p}$$

And so:

$$E_e = \frac{p^2}{2m_e} = \frac{Qm_p}{(m_e + m_p)}$$

$$E_e = \frac{0.783 \times 1.007}{5.49 \times 10^{-14} + 1.007}$$

$$E_e = 0.782\,57 \approx 0.783\,\text{MeV}$$

Thus, we should observe electrons with kinetic energies of about 0.783 MeV. In experiments, however, the electron has a **range** of energies from zero up to 0.783 MeV (Figure **12.27**) If the electron is not carrying 0.783 MeV of energy, where is the missing energy?

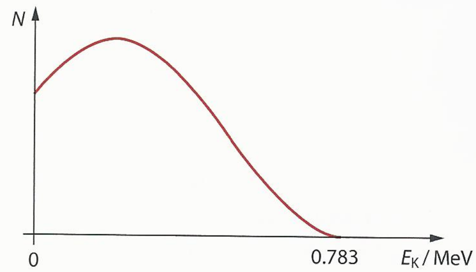


Figure 12.27 The number of electrons that carry a given energy as a function of energy.

Wolfgang Pauli hypothesised the existence of a third particle in the products of a beta decay in 1933. Since the energy of the electron in beta decay has a range of possible values, it means that a third very light particle must also be produced so that it carries the remainder of the available energy.

Enrico Fermi coined the word **neutrino** for the 'little neutral one'(Fermi is shown with Pauli and Heisenberg in Figure **12.28**).

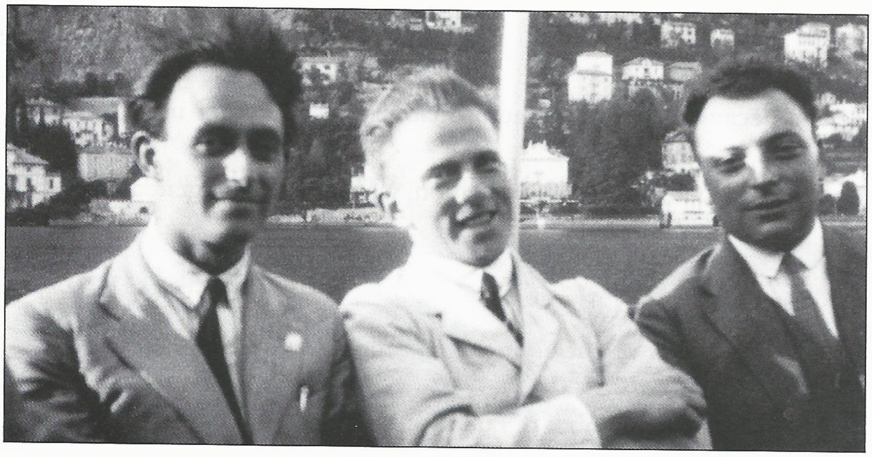


Figure 12.28 W. Pauli (right) , E. Fermi (left) either side of W. Heisenberg.

Research into neutrino physics is a great example of very large international research teams that work together and collaborate widely (by the Kamiokande and the super Kamiokande Japanese–American collaboration, the GALLEX and SAGE groups in Italy and Russia and the SNO (the Solar Neutrino Observatory) in Canada with Canadian, American and British participation).

As the neutrino is electrically neutral, it has no electromagnetic interactions. Its mass is negligibly small and so gravitational interactions are irrelevant. It is a lepton, so it does not have strong interactions. This leaves the weak interaction as the only interaction with which the neutrino can interact. This means that the neutrino can go through matter with very few interactions. In fact, about 10 billion neutrinos pass through your thumbnail every second, yet you do not feel a thing. For every 100 billion neutrinos that go through the Earth only one interacts with an atom in the Earth! Most of the neutrinos that arrive at Earth are produced in the Sun in the fusion reaction $p+p \rightarrow {}^{2}_{1}H+e^{+}+\nu_{e}$. Read the fascinating story of the solar neutrino problem in the Nature of science section at the end of this topic.

The radioactive decay law

As discussed in Topic **7**, the law of radioactive decay states that the rate of decay is proportional to the number of nuclei present that have not yet decayed:

$$\frac{dN}{dt} = -\lambda N$$

The constant of proportionality is denoted by λ and is called the **decay constant**. To see the meaning of the decay constant we argue as follows: in a short time interval dt the number of nuclei that will decay is $dN = \lambda N\, dt$ (we ignore the minus sign). The probability that any one nucleus will decay is therefore:

$$\text{probability} = \frac{dN}{N}$$

$$\text{probability} = \frac{N\lambda\, dt}{N}$$

$$\text{probability} = \lambda\, dt$$

and finally, the probability of decay per unit time is:

$$\frac{\text{probability}}{dt} = \lambda$$

The decay constant λ is the probability of decay per unit time.

The decay law is a differential equation, which when integrated gives:

$$N = N_0 e^{-\lambda t}$$

This is the number of nuclei present at time t given that the initial number (at $t=0$) is N_0.

As expected, the number of nuclei of the decaying element decreases exponentially as time goes on (Figure **12.29**).

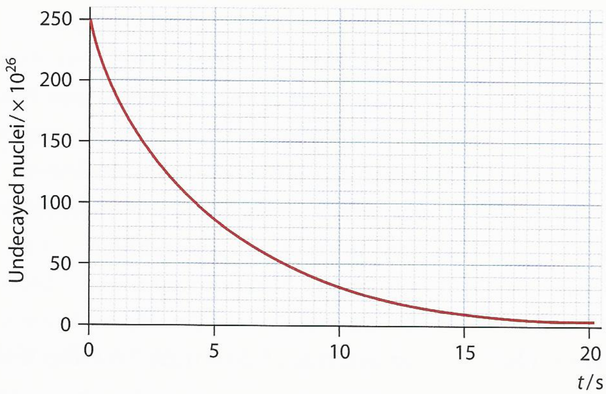


Figure 12.29 Radioactive decay follows an exponential decay law.

The (negative) rate of decay (i.e. the number of decays per second) is called **activity**, A:

$$A = -\frac{dN}{dt}$$

It follows from the exponential decay law that activity also satisfies an exponential law:

$$A = \lambda N_0 e^{-\lambda t}$$

Thus, the initial activity of a sample is given by the product of the decay constant and the number of atoms initially present, $A_0 = \lambda N_0$. Notice also that $A = \lambda N$.

After one half-life, $T_{\frac{1}{2}}$, half of the nuclei present have decayed and the activity has been reduced to half its initial value. So using either the formula for N or A (here we use the N formula):

$$\frac{N_0}{2} = N_0 e^{-\lambda t}$$

Taking logarithms we find:

$$\lambda T_{\frac{1}{2}} = \ln 2$$

$$\lambda T_{\frac{1}{2}} = 0.693$$

This is the relationship between the decay constant and the half-life.

Exam tip
It is important to know that the initial activity A_0 is λN_0.

Exam tip
A graph of A versus N gives a straight line whose slope is the decay constant.

Worked examples

12.18 Carbon-14 has a half-life of 5730 yr and in living organisms it has a decay rate of $0.25\,\text{Bq g}^{-1}$. A quantity of 20 g of carbon-14 was extracted from an ancient bone and its activity was found to be 1.81 Bq. What is the age of the bone?

Using the relationship between decay constant and half-life:

$$\lambda = \frac{\ln 2}{T_{\frac{1}{2}}}$$

$$\lambda = \frac{\ln 2}{5730}\,\text{yr}^{-1}$$

$$\lambda = 1.21 \times 10^{-4}\,\text{yr}^{-1}$$

When the bone was part of the living body the 20 g would have had an activity of $20 \times 0.25 = 5.0$ Bq. If the activity now is 1.81 Bq, then:

$$A = A_0 e^{-\lambda t}$$

$$1.81 = 5.0\, e^{-1.21 \times 10^{-4} t}$$

$$e^{-1.21 \times 10^{-4} t} = 0.362$$

$$-1.21 \times 10^{-4} t = -1.016$$

$$t = \frac{1.016}{1.21 \times 10^{-4}}$$

$$t \approx 8400\,\text{yr}$$

12.19 A container is filled with a quantity of a pure radioactive element X whose half-life is 5.0 minutes. Element X decays into a stable element Y. At time zero no quantity of element Y is present. Determine the time at which the ratio of atoms of Y to atoms of X is 5.

After time t the number of atoms of element X is given by $N = N_0 e^{-\lambda t}$. And the number of atoms of element Y is given by $N = N_0 - N_0 e^{-\lambda t}$. The decay constant is $\lambda = \frac{\ln 2}{5.0} = 0.1386\,\text{min}^{-1}$ and so we have that:

$$\frac{N_0 - N_0 e^{-0.1386t}}{N_0 e^{-0.1386t}} = 5$$

$$1 - e^{-0.1386t} = 5\, e^{-0.1386t}$$

$$e^{-0.1386t} = \frac{1}{6}$$

$$0.1386 \times t = 1.7981$$

$$t = 12.9\,\text{min}$$

Nature of science

The solar neutrino problem

In 1968, Ray Davis announced results of an experiment that tried to determine the number neutrinos arriving at Earth from the Sun. The idea was that the very rare interaction of neutrinos with ordinary chlorine would produce radioactive chlorine atoms that could then be detected, and hence the number of neutrinos determined. The results showed that the number of neutrinos was about one-third of what the theoretical calculation predicted. This created the 'solar neutrino problem'. There were three ways out: either the Davis experiment was wrong or the theory was wrong, or there was new physics involved.

Happily, it turned out that it was the last possibility that actually was in play. The number of neutrinos predicted by theory was based on the assumption that the neutrino was massless. If the neutrino had mass, then the theory would have to be modified because in that case 'neutrino oscillations' would take place. This is a rare quantum phenomenon, in which the three types of neutrinos could turn into each other. The Davis and subsequent experiments all measured electron neutrinos. Much later, when advances in instrumentation and computing power allowed experiments to detect all three types of neutrinos, the number was in agreement with the theory. But the neutrinos produced in the Sun were only electron neutrinos!

By this time, experiments in Japan and elsewhere provided convincing evidence that neutrinos had a tiny mass. So, because of neutrino oscillations, by the time the electron neutrinos reached Earth some of them had turned into muon neutrinos and some into tau neutrinos. On the average, about one-third would be electron neutrinos, in agreement with Davis's results! Ray Davis shared the 2002 Nobel prize in Physics.

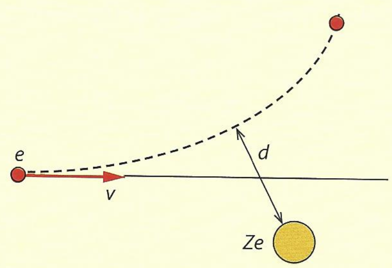
? Test yourself

24 An alpha particle is fired head-on at a stationary gold nucleus from far away. Calculate the initial speed of the particle so that the distance of closest approach is 8.5×10^{-15} m. (Take the mass of the alpha particle to be 6.64×10^{-27} kg.)

25 A particle of mass m and charge e is directed from very far away toward a massive ($M \gg m$) object of charge $+Ze$ with a velocity v, as shown in the diagram. The distance of closest approach is d. Sketch (on the same axes) a graph to show the variation with separation of:

a the particle's kinetic energy

b the particle's electric potential energy.



26 a Deviations from Rutherford scattering are expected when the alpha particles reach large energies. Suggest an explanation for this observation.

b Some alpha particles are directed at a thin foil of gold ($Z = 79$) and some others at a thin foil of aluminium ($Z = 13$). Initially, all alpha particles have the same energy. This energy is gradually increased. Predict in which case deviations from Rutherford scattering will first be observed.

27 Show that the nuclear density is the same for all nuclei. (Take the masses of the proton and neutron to be the same.)

28 a State the evidence in support of nuclear energy levels.

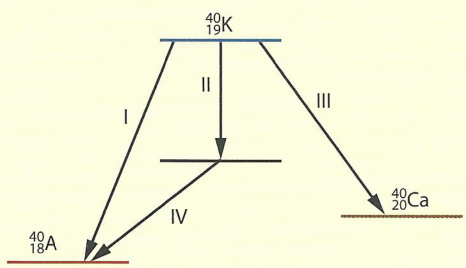
Radium's first excited nuclear level is 0.0678 MeV above the ground state.

b Write down the reaction that takes place when radium decays from the first excited state to the ground state.

c Find the wavelength of the photon emitted.

29 Plutonium ($^{242}_{94}$Pu) decays into uranium ($^{238}_{92}$U) by alpha decay. The energy of the alpha particles takes four distinct values: 4.90 MeV, 4.86 MeV, 4.76 MeV and 4.60 MeV. In all cases a gamma ray photon is also emitted except when the alpha energy is 4.90 MeV. Use this information to suggest a possible nuclear energy level diagram for uranium.

30 The diagram shows a few nuclear energy levels for $^{40}_{18}$Ar, $^{40}_{19}$K and $^{40}_{20}$Ca.



Identify the **four** indicated transitions.

31 **a** Find the decay constant for krypton-92, whose half-life is 3.00 s.

b Suppose that you start with $\frac{1}{100}$ mol of krypton. Estimate how many undecayed atoms of krypton there are after **i** 1 s, **ii** 2 s, **iii** 3 s.

32 **a** State the probability that a radioactive nucleus will decay during a time interval equal to a half-life.

b Calculate the probability that it will have decayed after the passage of three half-lives.

c A nucleus has not decayed after the passage of four half-lives. State the probability it will decay during the next half-life.

33 Estimate the activity of 1.0 g of radium-226 (molar mass = 226.025 g mol^{-1}). The half-life of radium-226 is 1600 yr.

34 The half-life of an unstable element is 12 days. Find the activity of a given sample of this element after 20 days, given that the initial activity was 3.5 MBq.

35 A radioactive isotope of half-life 6.0 days used in medicine is prepared 24 h prior to being administered to a patient. The activity must be 0.50 MBq when the patient receives the isotope. Estimate the number of atoms of the isotope that should be prepared.

36 The age of very old rocks can be found from uranium dating. Uranium is suitable because of its very long half-life: 4.5×10^9 yr. The final stable product in the decay series of uranium-238 is lead-206. Find the age of rocks that are measured to have a ratio of lead to uranium atoms of 0.80. You must assume that no lead was present in the rocks other than that due to uranium decaying.

37 The isotope $^{40}_{19}$K of potassium is unstable, with a half-life of 1.37×10^9 yr. It decays into the stable isotope $^{40}_{18}$Ar. Moon rocks were found to contain a ratio of potassium to argon atoms of 1 : 7. Find the age of the Moon rocks.

38 Two unstable isotopes are present in equal numbers (initially). Isotope A has a half-life of 4 min and isotope B has a half-life of 3 min. Calculate the ratio of the activity of A to that of B after: **a** 0 min, **b** 4 min, **c** 12 min.

39 A sample contains two unstable isotopes. A counter placed near it is used to record the decays. Discuss how you would determine each of the half-lives of the isotopes from the data.

40 The half-life of an isotope with a very long half-life cannot be measured by observing its activity as a function of time, since the variation in activity over any reasonable time interval would be too small to be observed. Let m be the mass in grams of a given isotope of long half-life.

a Show that the number of nuclei present in this quantity is $N_0 = \frac{m}{\mu} N_A$ where μ is the molar mass of the isotope in g mol^{-1} and N_A is the Avogadro constant.

b From $A = -\frac{dN}{dt} = N_0 \lambda e^{-\lambda t}$, show that the initial activity is $A_0 = \frac{mN_A}{\mu}\lambda$ and hence that the half-life can be determined by measuring the initial activity (in Bq) and the mass of the sample (in grams).