2.2 Forces

This section is an introduction to Newton's laws of motion. Classical physics is based to a great extent on these laws. It was once thought that knowledge of the present state of a system and all forces acting on it would enable the complete prediction of the state of that system in the future. This classical version of determinism has been modified partly due to quantum theory and partly due to chaos theory.

Learning objectives

- Treat bodies as point particles.
- Construct and interpret free-body force diagrams.
- Apply the equilibrium condition, $\Sigma F = 0$.
- Understand and apply Newton's three laws of motion.
- Solve problems involving solid friction.

Forces and their direction

A **force** is a vector quantity. It is important that we are able to correctly identify the **direction** of forces. In this section we will deal with the following forces.

Weight

This force is the result of the gravitational attraction between the mass m of a body and the mass of the planet on which the body is placed. The **weight** of a body is given by the formula:

$$W = mg$$

where m is the mass of the body and g is gravitational field strength of the planet (Subtopic **6.2**). The unit of g is newton per kilogram, $\mathrm{N\,kg^{-1}}$. The gravitational field strength is also known as 'the acceleration due to gravity' or the 'acceleration of free fall'. Therefore the unit of g is also $\mathrm{m\,s^{-2}}$.

If m is in kg and g in $\mathrm{N\,kg^{-1}}$ or $\mathrm{m\,s^{-2}}$ then W is in newtons, N. On the **surface** of the Earth, $g = 9.81\,\mathrm{N\,kg^{-1}}$ – a number that we will often approximate by the more convenient $10\,\mathrm{N\,kg^{-1}}$. This force is always directed vertically downward, as shown in Figure **2.24**.

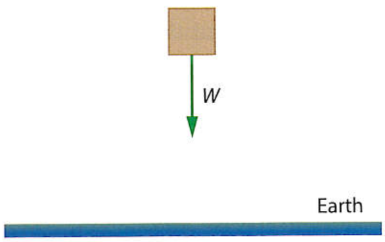


Figure 2.24 The weight of an object is always directed vertically downward.

The mass of an object is the same everywhere in the universe, but its weight depends on the **location** of the body. For example, a mass of 70 kg has a weight of 687 N on the surface of the Earth ($g = 9.81\,\mathrm{N\,kg^{-1}}$) and a weight of 635 N at a height of 250 km from the Earth's surface (where $g = 9.07\,\mathrm{N\,kg^{-1}}$). However, on the surface of Venus, where the gravitational field strength is only $8.9\,\mathrm{N\,kg^{-1}}$, the weight is 623 N.

Tension

The force that arises in any body when it is stretched is called **tension**. A string that is taut is said to be under tension. The tension force is the result of electromagnetic interactions between the molecules of the material making up the string. A tension force in a string is created when two forces are applied in opposite directions at the ends of the string (Figure **2.25**).

Figure 2.25 A tension force in a string.

To say that there is tension in a string means that an arbitrary point on the string is acted upon by two forces (the tension T) as shown in Figure **2.26**. If the string hangs from a ceiling and a mass m is tied at the other end, tension develops in the string. At the point of support at the ceiling, the tension force pulls down on the ceiling and at the point where the mass is tied the tension acts upwards on the mass.

In most cases we will idealise the string by assuming it is massless. This does not mean that the string really is massless, but rather that its mass is so small compared with any other masses in the problem that we can neglect it. In that case, the tension T is the same at all points on the string. The direction of the tension force is along the string. Further examples of tension forces in a string are given in Figure **2.27**. A string or rope that is not taut has zero tension in it.

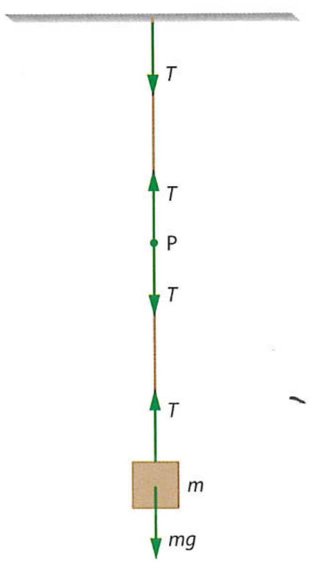


Figure 2.26 The tension is directed along the string.

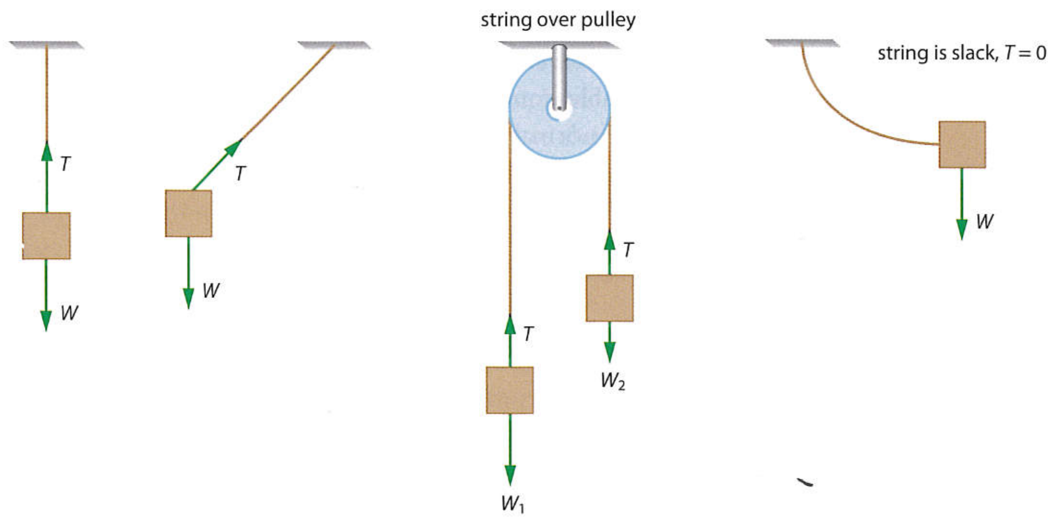


Figure 2.27 More examples of tension forces.

Forces in springs

A spring that is pulled so that its length increases will develop a tension force inside the spring that will tend to bring the length back to its original value. Similarly, if it is compressed a tension force will again try to restore the length of the spring, Figure **2.28**. Experiments show that for a range of extensions of the spring, the tension force is proportional to the extension, $T = kx$, where k is known as the spring constant. This relation between tension and extension is known as **Hooke's law**.

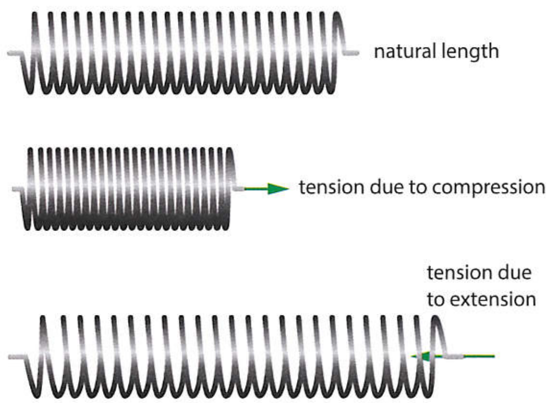


Figure 2.28 Tension forces in a spring.

Normal reaction contact forces

If a body touches another body, there is a **force of reaction** or **contact force** between the two bodies. This force is perpendicular to the surface of the body exerting the force. Like tension, the origin of this force is also electromagnetic. In Figure **2.29** we show the reaction force on several bodies.

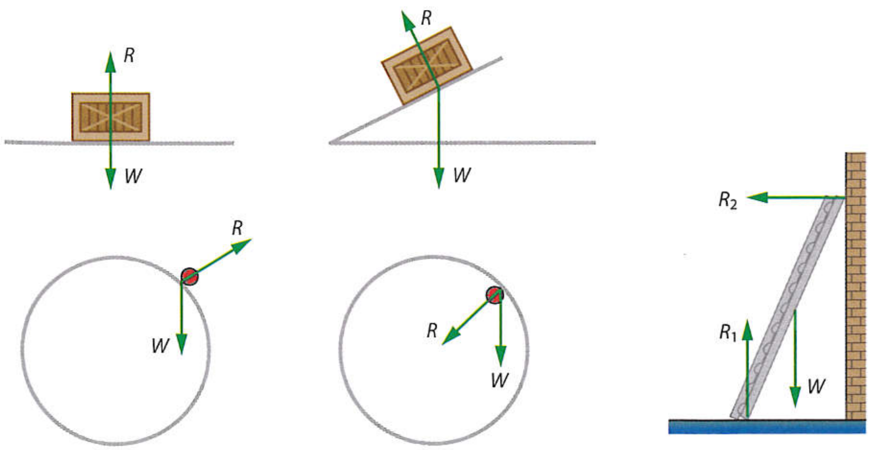


Figure 2.29 Examples of reaction forces, R.

We can understand the existence of contact reaction forces in a simple model in which atoms are connected by springs. The block pushes down on the atoms of the table, compressing the springs under the block (Figure **2.30**). This creates the normal reaction force on the block.

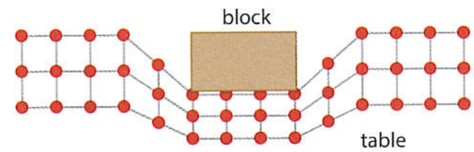


Figure 2.30 A simple model of contact forces.

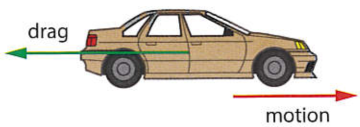


Figure 2.31 The drag force on a moving car.

Drag forces

Drag forces are forces that oppose the motion of a body through a fluid (a gas or a liquid). Typical examples are the air resistance force experienced by a car (Figure **2.31**) or plane, or the resistance force experienced by a steel marble dropped into a jar of honey. Drag forces are directed opposite to the velocity of the body and in general depend on the speed and shape of the body. The higher the speed, the higher the drag force.

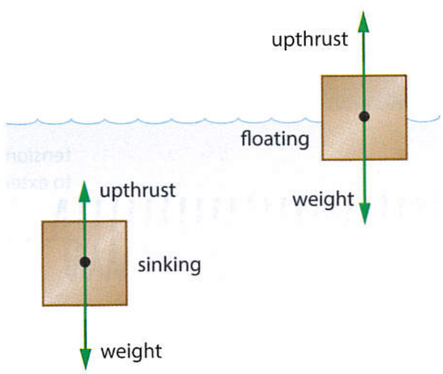


Figure 2.32 Upthrust.

Upthrust

Any object placed in a fluid experiences an upward force called **upthrust** (Figure **2.32**). If the upthrust force equals the weight of the body, the body will float in the fluid. If the upthrust is less than the weight, the body will sink. Upthrust is caused by the pressure that the fluid exerts on the body.

Frictional forces

Frictional forces generally oppose the motion of a body (Figure **2.33**). These forces are also electromagnetic in origin.

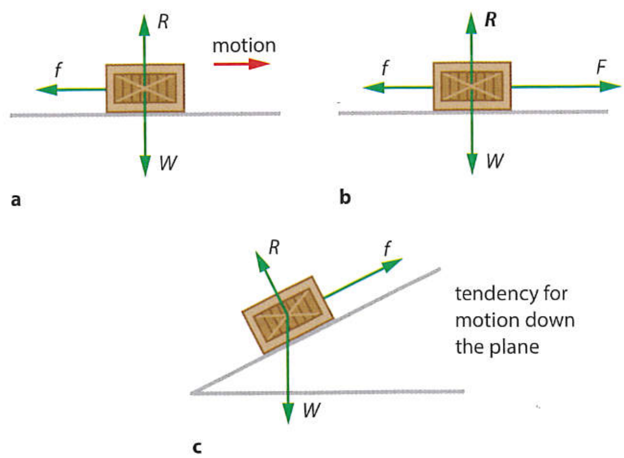


Figure 2.33 Examples of frictional forces, *f*. In **a** there is motion to the right, which is opposed by a single frictional force that will eventually stop the body. In **b** the force accelerating the body is opposed by a frictional force. In **c** the body does not move; but it does have a tendency to move down the plane and so a frictional force directed up the plane opposes this tendency, keeping the body in equilibrium.

Friction arises whenever one body slides over another. In this case we have **dynamic or kinetic friction**. Friction also arises whenever there is a tendency for motion, not necessarily motion itself. For example a block that rests on an inclined plane has a tendency to slide down the plane, so there is a force of friction up the plane. Similarly, if you pull on a block on a level rough road with a small force the block will not move. This is because a force of friction develops that is equal and opposite to the pulling force. In this case we have **static friction**.

In the simple model of matter consisting of atoms connected by springs, pushing the block to the right results in springs stretching and compressing. The net result is a force opposing the motion: friction (Figure **2.34**).

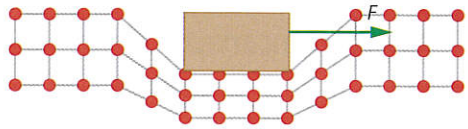


Figure 2.34 Friction in the simple atoms-and-springs model of matter.

A more realistic model involves irregularities (called **asperities**) in the surfaces which interlock, opposing sliding, as shown in Figure **2.35**.

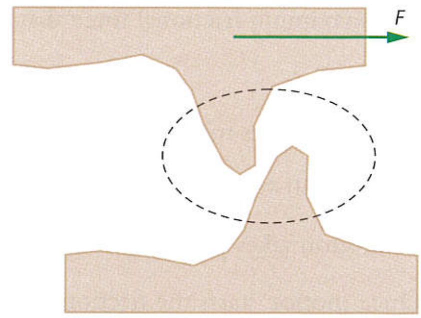


Figure 2.35 Exaggerated view of how asperities oppose the sliding of one surface over the other.

Frictional forces are still not very well understood and there is no theory of friction that follows directly from the fundamental laws of physics. However, a number of simple, empirical 'laws' of friction have been discovered. These are not always applicable and are only approximately true, but they are useful in describing frictional forces in general terms.

These so-called **friction laws** may be summarised as follows:

- The area of contact between the two surfaces does not affect the frictional force.
- The force of dynamic friction is equal to:
  $f_d = \mu_d R$
  where R is the normal reaction force between the surfaces and μ_d is the **coefficient of dynamic friction**.
- The force of dynamic friction does not depend on the speed of sliding.
- The **maximum** force of static friction that can develop between two surfaces is given by:
  $f_s = \mu_s R$
  where R is the normal reaction force between the surfaces and μ_s is the **coefficient of static friction**, with $\mu_s > \mu_d$.

Exam tip
One of the most common mistakes is to think that $\mu_s R$ is the formula that gives the static friction force. This is not correct. This formula gives the maximum possible static friction force that can develop between two surfaces.

Figure **2.36** shows how the frictional force f varies with a pulling force F. The force F pulls on a body on a horizontal rough surface. Initially the static frictional force matches the pulling force and we have no motion, $f_s = F$. When the pulling force exceeds the maximum possible static friction force, $\mu_s R$, the frictional force drops abruptly to the dynamic value of $\mu_d R$ and stays at that constant value as the object accelerates. This is a well-known phenomenon of everyday life: it takes a lot of force to get a heavy piece of furniture to start moving (you must exceed the maximum value of the static friction force), but once you get it moving, pushing it along becomes easier (you are now opposed by the smaller dynamic friction force).

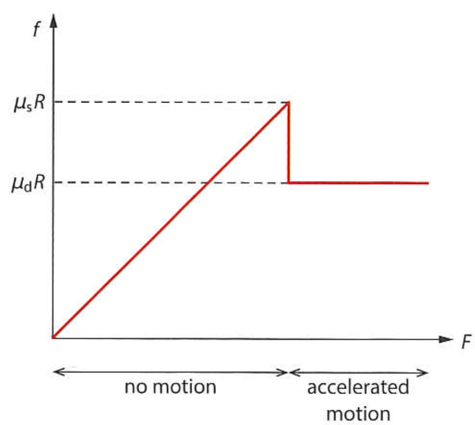


Figure 2.36 The variation of the frictional force f between surfaces with the pulling force F.

Worked example

2.17 A brick of weight 50 N rests on a horizontal surface. The coefficient of static friction between the brick and the surface is 0.60 and the coefficient of dynamic friction is 0.20. A horizontal force F is applied to the brick, its magnitude increasing uniformly from zero. Once the brick starts moving the pulling force no longer increases. Estimate the net force on the moving brick.

The maximum frictional force that can develop between the brick and the surface is:

$f_s = \mu_s R$

which evaluates to:

$0.60 \times 50 = 30\,\text{N}$

So motion takes place when the pulling force is just barely larger than 30 N.

Once motion starts the frictional force will be equal to $\mu_d R$, i.e.

$0.20 \times 50 = 10\,\text{N}$

The net force on the brick in that case will be just larger than $30 - 10 = 20\,\text{N}$.

Free-body diagrams

A **free-body diagram** is a diagram showing the magnitude and direction of all the forces acting on a chosen body. The body is shown on its own, free of its surroundings and of any other bodies it may be in contact with. We treat the body as a **point particle**, so that all forces act through the same point. In Figure **2.37** we show three situations in which forces are acting; below each is the corresponding free-body diagram for the coloured bodies.

In any mechanics problem, it is important to be able to draw correctly the free-body diagrams for all the bodies of interest. It is also important that the length of the arrow representing a given force is proportional to the magnitude of the force.

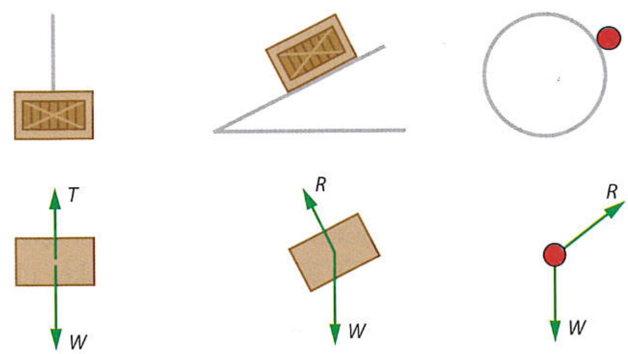


Figure 2.37 Free-body diagrams for the coloured bodies.

Newton's first law of motion

Suppose you have two identical train carriages. Both are equipped with all the apparatus you need to do physics experiments. One train carriage is at rest at the train station. The other moves in a straight line with constant speed – the ride is perfectly smooth, there are no bumps, there is no noise and there are no windows to look outside. Every physics experiment conducted in the train at rest will give identical results to similar experiments made in the moving train. We have no way of determining whether a carriage is 'really at rest' or 'really moving'. We find it perfectly natural to believe, correctly, that no net force is present in the case of the carriage at rest. Therefore no net force is required in the case of the carriage moving in a straight line with constant speed.

Newton's first law (with a big help from Galileo) states that:

> **When the net force on a body is zero, the body will move with constant velocity (which may be zero).**

In effect, Newton's first law defines what a force is. A force is what changes a body's velocity. A force is *not* what is required to keep something moving, as Aristotle thought.

Using the law in reverse allows us to conclude that if a body is not moving with constant velocity (which may mean not moving in a straight line, or not moving with constant speed, or both) then a force must be acting on the body. So, since the Earth revolves around the Sun we know that a force must be acting on the Earth.

Newton's first law is also called the law of **inertia**. Inertia is what keeps the body in the same state of motion when no forces act on the body. When a car accelerates forward, the passengers are thrown back into their seats because their original state of motion was motion with low speed. If a car brakes abruptly, the passengers are thrown forward (Figure **2.38**). This implies that a mass tends to stay in the state of motion it was in before the force acted on it. The reaction of a body to a change in its state of motion (acceleration) is inertia.

Figure 2.38 The car was originally travelling at high speed. When it hits the wall the car stops but the passenger stays in the original high speed state of motion. This results in the crash dummy hitting the steering wheel and the windshield (which is why it is a good idea to have safety belts and air bags).

Newton's third law of motion

Newton's third law states that if body A exerts a force on body B, then body B will exert an equal and opposite force on body A. These forces are known as **force pairs**. Make sure you understand that these equal and opposite forces act on different bodies. Thus, you cannot use this law to claim that it is impossible to ever have a net force on a body because for every force on it there is also an equal and opposite force. Here are a few examples of this law:

- You stand on roller skates facing a wall. You push on the wall and you move away from it. This is because you exerted a force on the wall and in turn the wall exerted an equal and opposite force on you, making you move away (Figure **2.39**).

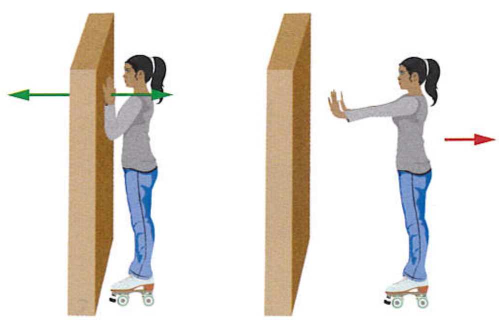


Figure 2.39 The girl pushes on the wall so the wall pushes on her in the opposite direction.

- You step on the bathroom scales. The scales exert an upward force on you and so you exert a downward force on the scales. This is the force shown on the scales (Figure **2.40**).
- A helicopter hovers in air (Figure **2.41**). Its rotors exert a force downward on the air. Thus, the air exerts the upward force on the helicopter that keeps it from falling.
- A book of mass 2 kg is allowed to fall feely. The Earth exerts a force on the book, namely the weight of the book of about 20 N. Thus, the book exerts an equal and opposite force on the Earth – a force upward equal to 20 N.

Figure 2.40 The familiar bathroom scales do not measure mass. They measure the force that you exert on the scales. This force is equal to the weight only when the scales are at rest.

Figure 2.41 The upward force on the rotor is due to the force the rotor exerts on the air downward.

You must be careful with situations in which two forces are equal and opposite; they do not always have to do with the third law. For example, a block of mass 3 kg resting on a horizontal table has two forces acting on it – its weight of about 30 N and the normal reaction force from the table that is also 30 N. These two forces are equal and opposite, but they are acting on the same body and so have nothing to do with Newton's third law. (We have seen in the last bullet point above the force that pairs with the weight of the block. The force that pairs with the reaction force is a downward force on the table.)

Newton's third law also applies to cases where there is no contact between the bodies. Examples are the **electric** force between two electrically charged particles or the **gravitational** force between any two massive particles. These forces must be equal and opposite (Figure **2.42**).

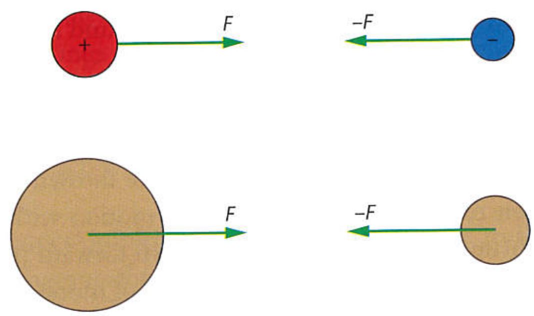


Figure 2.42 The two charges and the two masses are different, but the forces are equal and opposite.

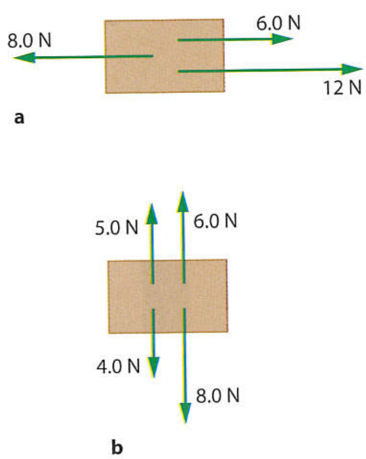


Figure 2.43 The net force is found by plain addition and/or subtraction when the forces are in the same or opposite direction.

Equilibrium

Equilibrium of a point particle means that the **net force** on the particle is zero. The net force on a particle is the one single force whose effect is the same as the combined effect of individual forces acting on the particle. We denote it by ΣF. Finding the net force is easy when the forces are in the same or opposite directions (Figure **2.43**).

In Figure **2.43a**, the net force is (if we take the direction to the right to be positive) $\Sigma F = 12 + 6.0 - 8.0 = 10$ N. This is positive, indicating a direction to the right.

In Figure **2.43b**, the net force is (we take the direction upward to be positive) $\Sigma F = 5.0 + 6.0 - 4.0 - 8.0 = -1.0$ N. The negative sign indicates a direction vertically down.

Worked example

2.18 Determine the magnitude of the force F in Figure **2.44**, given that the block is in equilibrium.

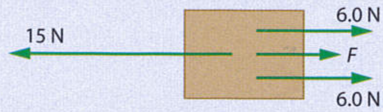


Figure 2.44

For equilibrium, $\Sigma F = 0$, and so:

$$6.0 + F + 6.0 - 15 = 0$$

This gives $F = 3.0\,\text{N}$.

Solving equilibrium problems

When there are angles between the various forces, solving equilibrium problems will involve finding components of forces using vector methods. We choose a set of axes whose origin is the body in question and find the components of all the forces on the body. Figure **2.45** shows three forces acting at the same point. We have equilibrium, which means the net force acting at the point is zero. We need to find the unknown magnitude and direction of force F_1. This situation could represent three people pulling on three ropes that are tied at a point.

Finding components along the horizontal (x) and vertical (y) directions for the known forces F_2 and F_3, we have:

$F_{2x} = 0$

$F_{2y} = -22.0\,\text{N}$ (add minus sign to show the direction)

$F_{3x} = -29.0\cos 37° = -23.16\,\text{N}$ (add minus sign to show the direction)

$F_{3y} = 29.0\,\text{sins}\,37° = 17.45\,\text{N}$

Equilibrium demands that $\Sigma F_x = 0$ and $\Sigma F_y = 0$.

$\Sigma F_x = 0$ implies:

$$F_{1x} + 0 - 23.16 = 0 \Rightarrow F_{1x} = 23.16\,\text{N}$$

$\Sigma F_y = 0$ implies:

$$F_{1y} - 22.0 + 17.45 = 0 \Rightarrow F_{1y} = 4.55\,\text{N}$$

Therefore, $F_1 = \sqrt{23.16^2 + 4.55^2} = 23.6\,\text{N}$

The angle is found from $\tan\theta = \dfrac{F_{1y}}{F_{1x}} \Rightarrow \theta = \tan^{-1}\left(\dfrac{4.55}{23.16}\right) = 11.1°$

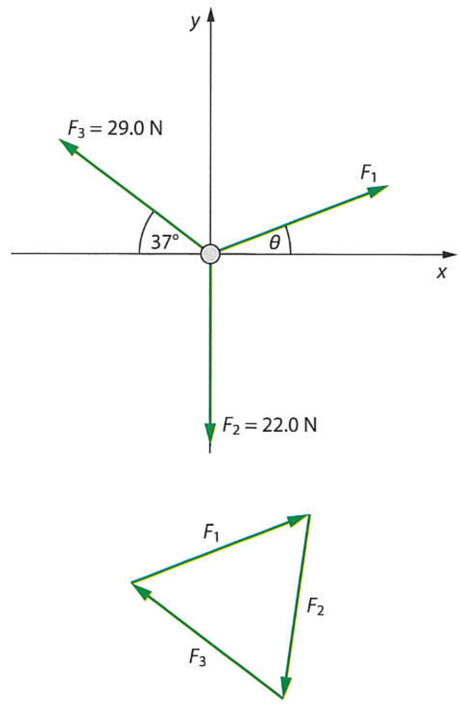


Figure 2.45 Force diagram of three forces in equilibrium pulling a common point. Notice that the three vectors representing the three forces form a triangle.

Exam tip

If we know the x- and y-components of a force we can find the magnitude of the force from $F = \sqrt{F_x^2 + F_y^2}$.

Worked example

2.19 A body of weight 98.0 N hangs from two strings that are attached to the ceiling as shown in Figure **2.46**. Determine the tension in each string.

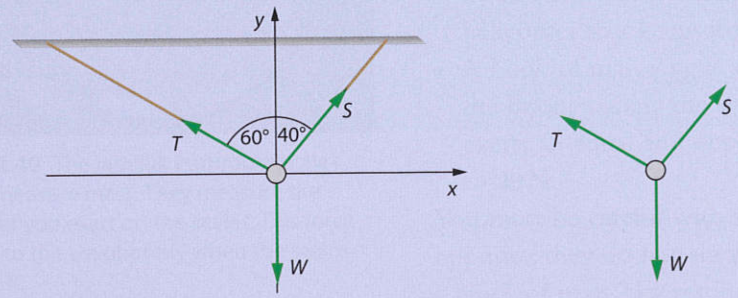


Figure 2.46

The three forces acting on the body are as shown, with T and S being the tensions in the two strings and W its weight. Taking components about horizontal and vertical axes through the body we find:

$T_x = -T\cos 30°$ (add minus sign to show the direction) $S_x = S\cos 50°$ $W_x = 0$

$T_y = T\sin 30°$ $S_y = S\sin 50°$ $W_y = -98.0\,\text{N}$

Equilibrium thus demands $\Sigma F_x = 0$ and $\Sigma F_y = 0$.

$\Sigma F_x = 0$ implies:

$$-T\cos 30° + S\cos 50° = 0$$

$\Sigma F_y = 0$ implies:

$$T\sin 30° + S\sin 50° - 98.0 = 0$$

From the first equation we find that:

$$S = T\frac{\cos 30°}{\cos 50°} = 1.3473 \times T$$

Substituting this in the second equation gives:

$$T(\sin 30° + 1.3473\sin 50°) = 98$$

which solves to give:

$$T = 63.96 \approx 64.0\,\text{N}$$

Hence $S = 1.3473 \times 63.96 = 86.17 \approx 86.2\,\text{N}$.

2.20 A mass of 125 g is attached to a spring of spring constant $k = 58\,\mathrm{N\,m^{-1}}$ that is hanging vertically.

a Find the extension of the spring.

b If the mass and the spring are placed on the Moon, will there be any change in the extension of the spring?

a The forces on the hanging mass are its weight and the tension in the spring. By Hooke's law, the tension in the spring is kx, where x is the extension and k the spring constant. Since we have equilibrium, the two forces are equal in magnitude. Therefore:

$$kx = mg$$

$$x = \frac{mg}{k}$$

$$x = \frac{0.125 \times 10}{58} \quad \text{(taking } g = 10\,\mathrm{N\,kg^{-1}}\text{)}$$

$$x = 0.022\,\mathrm{m}$$

The extension is 2.2 cm.

b The extension will be less, since the acceleration of gravity is less.

Newton's second law of motion

Newton's second law states that:

> The net force on a body of constant mass is proportional to that body's acceleration and is in the same direction as the acceleration.

Mathematically:

$$F = ma$$

where the constant of proportionality, m, is the *mass* of the body.

Figure **2.47** shows the net force on a freely falling body, which happens to be its weight, $W = mg$. By Newton's second law, the net force equals the mass times the acceleration, and so:

$$mg = ma$$

$$a = g$$

That is, the acceleration of the freely falling body is exactly g. Experiments going back to Galileo show that indeed all bodies fall with the same acceleration in a vacuum (the acceleration of free fall) irrespective of their density, their mass, their shape and the material from which they are made. Look for David Scott's demonstration dropping a hammer and feather on the Moon in Apollo 15's mission in 1971. You can do the same demonstration without going to the Moon by placing a hammer and a

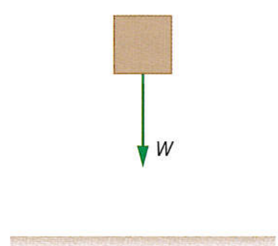


Figure 2.47 A mass falling to the ground acted upon by gravity.

Exam tip
To solve an '$F = ma$' problem:

- Make a diagram.
- Identify the forces on the body of interest.
- Find the net force on each body, taking the direction of acceleration to be the positive direction.
- Apply $F_{net} = ma$ to each body.

feather on a book and dropping the book. If the heavy and the light object fell with different accelerations the one with the smaller acceleration would lift off the book – but it doesn't.

The equation $F = ma$ defines the unit of force, the newton (N). One newton is the force required to accelerate a mass of 1 kg by $1\,\text{m}\,\text{s}^{-2}$ in the direction of the force.

It is important to realise that the force in the second law is the net force ΣF on the body.

Worked examples

2.21 A man of mass $m = 70\,\text{kg}$ stands on the floor of an elevator. Find the force of reaction he experiences from the elevator floor when the elevator:

a is standing still
b moves up at constant speed $3.0\,\text{m}\,\text{s}^{-1}$
c moves up with acceleration $4.0\,\text{m}\,\text{s}^{-2}$
d moves down with acceleration $4.0\,\text{m}\,\text{s}^{-2}$
e moves down, slowing down with deceleration $4.0\,\text{m}\,\text{s}^{-2}$.
Take $g = 10\,\text{m}\,\text{s}^{-2}$.

Two forces act on the man: his weight mg vertically down and the reaction force R from the floor vertically up.

a There is no acceleration and so by Newton's second law the net force on the man must be zero. Hence:

$$R = mg$$
$$R = 7.0 \times 10^2\,\text{N}$$

b There is no acceleration and so again:

$$R = mg$$
$$R = 7.0 \times 10^2\,\text{N}$$

c There is acceleration upwards. The net force in the direction of the acceleration is given by:

$$\Sigma F = R - mg$$
So: $ma = R - mg$
$$\Rightarrow \quad R = mg + ma$$
$$R = 700\,\text{N} + 280\,\text{N}$$
$$R = 9.8 \times 10^2\,\text{N}$$

d We again have acceleration, but this time in the downward direction. We need to find the net force in the direction of the acceleration:

$$\Sigma F = mg - R$$
So: $ma = mg - R$
$$\Rightarrow \quad R = mg - ma$$
$$R = 700\,\text{N} - 280\,\text{N}$$
$$R = 4.2 \times 10^2\,\text{N}$$

e The deceleration is equivalent to an upward acceleration, so this case is identical to part **c**.

2.22 A man of mass 70 kg is standing in an elevator. The elevator is moving **upward** at a speed of $3.0\,\mathrm{m\,s^{-1}}$. The elevator comes to rest in a time of 2.0 s. Determine the reaction force on the man from the elevator floor during the period of deceleration.

Use $a = v - \frac{u}{t}$ to find the acceleration experienced by the man:

$$a = -\frac{3.0}{2.0} = -1.5\,\mathrm{m\,s^{-2}}$$

The minus sign shows that this acceleration is directed in the **downward** direction. So we must find the net force in the down direction, which is $\Sigma F = mg - R$. (We then use the **magnitude** of the accelerations, as the form of the equation takes care of the direction.)

$$ma = mg - R$$

$$\Rightarrow \quad R = mg - ma$$

$$R = 700 - 105$$

$$R = 595 \approx 6.0 \times 10^2\,\mathrm{N}$$

If, instead, the man was moving **downward** and then decelerated to rest, the acceleration is directed upward and $\Sigma F = R - mg$.

So: $ma = R - mg$

$$\Rightarrow \quad R = mg + ma$$

$$R = 700 + 105$$

$$R = 805 \approx 8.0 \times 10^2\,\mathrm{N}$$

Both cases are easily experienced in daily life. When the elevator goes up and then stops we feel 'lighter' during the deceleration period. When going down and about to stop, we feel 'heavier' during the deceleration period. The feeling of 'lightness' or 'heaviness' has to do with the reaction force we feel from the floor.

2.23 **a** Two blocks of mass 4.0 kg and 6.0 kg are joined by a string and rest on a frictionless horizontal table (Figure **2.48**). A force of 100 N is applied horizontally on one of the blocks. Find the acceleration of each block and the tension in the string.

b The 4.0 kg block is now placed on top of the other block. The coefficient of static friction between the two blocks is 0.45. The bottom block is pulled with a horizontal force F. Calculate the magnitude of the maximum force F that will result in both blocks moving together without slipping.

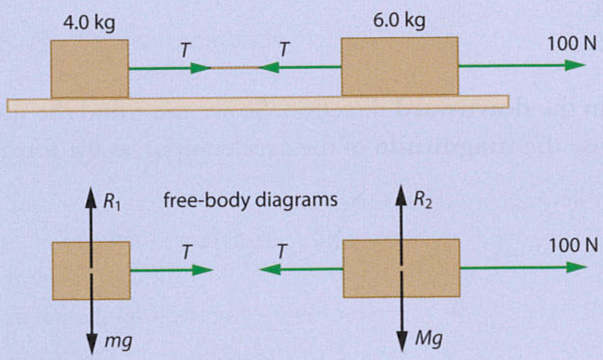


Figure 2.48

a This can be done in **two** ways.

Method 1

Let the acceleration of the system be a. The net horizontal force on the 6.0 kg mass is $100 - T$ and the net horizontal force on the 4.0 kg mass is just T. Thus, applying Newton's second law separately on each mass:

$$100 - T = 6.0a$$

$$T = 4.0a$$

Solving for a (by adding the two equations) gives:

$$100 = 10a$$

$$\Rightarrow \quad a = 10\,\text{m}\,\text{s}^{-2}$$

The tension in the string is therefore:

$$T = 4.0 \times 10$$

$$T = 40\,\text{N}$$

Note: The free-body diagram makes it clear that the 100 N force acts only on the body to the right. It is a common mistake to say that the body to the left is also acted upon by the 100 N force.

Method 2

We may consider the two bodies as one of mass 10 kg. The net force on the body is 100 N. Note that the tensions are irrelevant now since they cancel out. (They did not in Method 1, as they acted on different bodies. Now they act on the same body. They are now **internal** forces and these are irrelevant.)

Applying Newton's second law on the single body we have:

$$100 = 10a$$

$$\Rightarrow \quad a = 10\,\text{m s}^{-2}$$

But to find the tension we must break up the combined body into the original two bodies. Newton's second law on the 4.0 kg body gives:

$$T = 4a = 40\,\text{N}$$

(the tension on this block is the net force on the block). If we used the other block, we would see that the net force on it is $100 - T$ and so:

$$100 - T = 6 \times 10 = 60\,\text{N}$$

This gives $T = 40$ N, as before.

b If the blocks move together they must have the same acceleration. Treating the two blocks as one (of mass 10 kg), the acceleration will be $a = \frac{F}{10}$ (Figure **2.49a**).

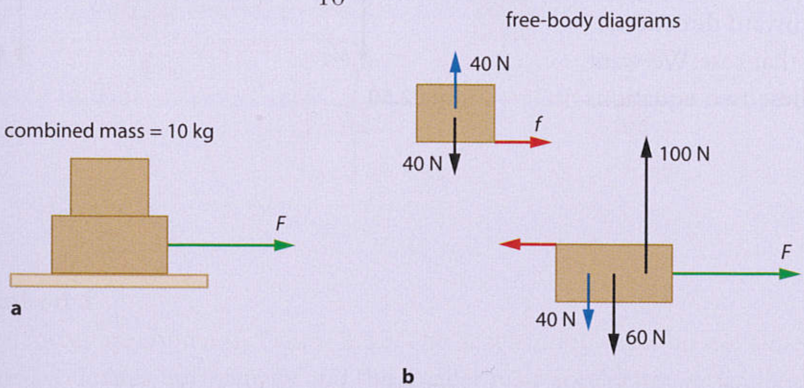


Figure 2.49 **a** Treating the blocks as one. **b** The free-body diagram for each block.

The forces on each block are shown in Figure **2.49b**. The force pushing the smaller block forward is the frictional force f that develops between the blocks. The **maximum** value f can take is:

$$f = \mu_s R = 0.45 \times 40 = 18\,\text{N}$$

So the acceleration of the small block is:

$$a = \frac{18}{4.0} = 4.5\,\text{m s}^{-2}$$

But $a = \frac{F}{10}$, so:

$$\frac{F}{10} = 4.5\,\text{m s}^{-2}$$

$$\Rightarrow \quad F = 45\,\text{N}$$

2.24 Two masses of $m = 4.0\,\text{kg}$ and $M = 6.0\,\text{kg}$ are joined together by a string that passes over a pulley (this arrangement is known as Atwood's machine). The masses are held stationary and suddenly released. Determine the acceleration of each mass.

Intuition tells us that the larger mass will start moving downward and the small mass will go up. So if we say that the larger mass's acceleration is a, then the other mass's acceleration will also be a in magnitude but, of course, in the opposite direction. The two accelerations are the same because the string cannot be extended.

Method 1

The forces on each mass are weight mg and tension T on m and weight Mg and tension T on M (Figure **2.50**).

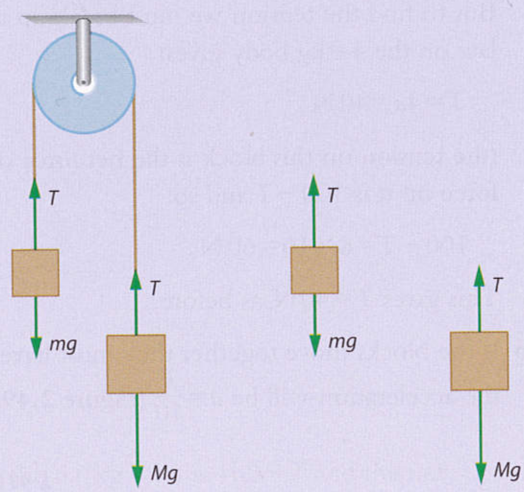


Figure 2.50

Newton's second law applied to each mass gives:

$$T - mg = ma \quad (1)$$

$$Mg - T = Ma \quad (2)$$

Note these equations carefully. Each says that the net force on the mass in question is equal to that mass times that mass's acceleration. In the first equation, we find the net force in the upward direction, because that is the direction of acceleration. In the second, we find the net force in downward direction, since that is the direction of acceleration in that case. We want to find the acceleration, so we simply add these two equations to find:

$$Mg - mg = (m + M)a$$

Hence:

$$a = \frac{M - m}{M + m} g$$

(Note that if $M \gg m$ the acceleration tends to g. Can you think why this is?) This shows clearly that if the two masses are equal, then there is no acceleration. This is a convenient method for measuring g. Atwood's machine effectively 'slows down' g so the falling mass has a much smaller acceleration from which g can then be determined. Putting in the numbers for our example we find $a = 2.0\,\text{m}\,\text{s}^{-2}$.

Having found the acceleration we may, if we wish, also find the tension in the string, T. Putting the value for a in formula (1) we find:

$$T = m\left(\frac{M - m}{M + m}\right)g + mg$$

$$T = 2\left(\frac{Mm}{M + m}\right)g$$

(If $M \gg m$ the tension tends to $2mg$. Can you see why?)

Method 2

We treat the two masses as one body and apply Newton's second law on this body (but this is trickier than in the previous example) – see Figure **2.51**.

In this case the net force is $Mg - mg$ and, since this force acts on a body of mass $M + m$, the acceleration is found as before from $F = \text{mass} \times \text{acceleration}$. Note that the tension T does not appear, as it is now an internal force.

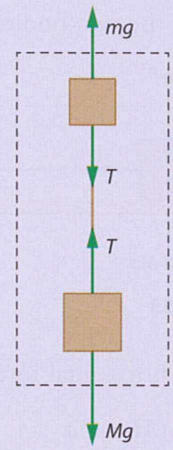


Figure 2.51

2.25 In Figure **2.52**, a block of mass M is connected to a smaller mass m through a string that goes over a pulley. Ignoring friction, find the acceleration of each mass and the tension in the string.

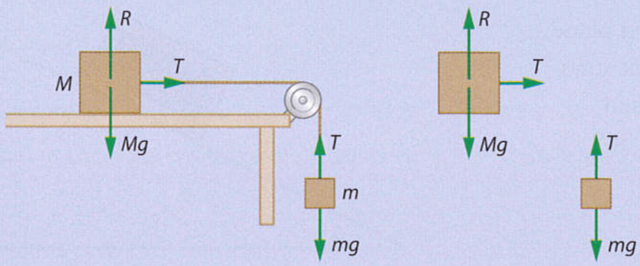


Figure 2.52

Method 1

The forces are shown in Figure **2.52**. The acceleration must be the same magnitude for both masses, but the larger mass accelerates horizontally and the smaller mass accelerates vertically downwards. The free-body diagrams on the right show the forces on the individual masses. Taking each mass separately:

$mg - T = ma$ (small mass accelerating downwards)

$T = Ma$ (large mass accelerating horizontally to the right)

Adding the two equations, we get:

$$mg = ma + Ma$$

$$\Rightarrow \quad a = \frac{mg}{M+m}$$

(If $M \gg m$ the acceleration tends to zero. Why?)

From the expression for T for the larger mass, we have:

$$T = Ma = \frac{Mmg}{M+m}$$

Method 2

Treating the two bodies as one results in the situation shown in Figure **2.53**.

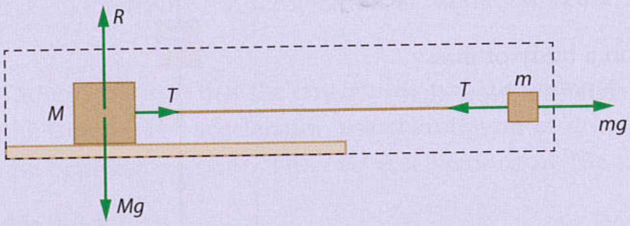


Figure 2.53

The net horizontal force on the combined mass $M+m$ is mg. Hence:

$$mg=(M+m)a$$

$$\Rightarrow \quad a=\frac{mg}{M+m}$$

The tension can then be found as before.

2.26 A block of mass 2.5 kg is held on a rough inclined plane, as shown in Figure **2.54**. When released, the block stays in place. The angle of the incline is slowly increased and when the angle becomes slightly larger than 38° the block begins to slip down the plane.

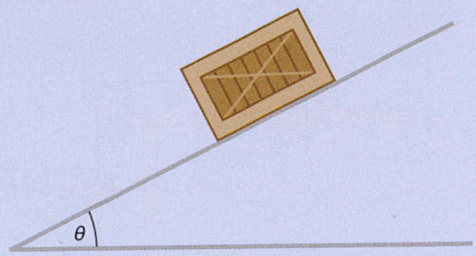


Figure 2.54

a Calculate the coefficient of static friction between the block and the inclined plane.

b The angle of the incline is increased to 49°. The coefficient of dynamic friction between the block and the incline is 0.26. Calculate the force that must be applied to the block along the plane so it moves up the plane with an acceleration of $1.2\,\mathrm{m\,s^{-2}}$.

a The forces on the block just before slipping are shown in Figure **2.55**. The frictional force is f and the normal reaction is R. The components of the weight are $mg\sin\theta$ down the plane and $mg\cos\theta$ at right angles to the plane.

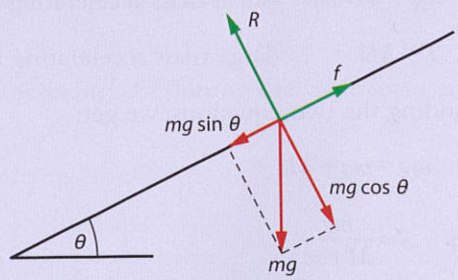


Figure 2.55

Because the block is about to slip, the frictional force is the maximum possible static frictional force and so $f=\mu_s R$. Equilibrium demands that:

$$mg\sin\theta=f$$

$$mg\cos\theta=R$$

Divide the first equation by the second to get:

$$\tan\theta = \frac{f}{R}$$

Now use the fact that $f = \mu_s R$ to find:

$$\tan\theta = \frac{\mu_s R}{R}$$

$$\tan\theta = \mu_s$$

Hence $\mu_s = \tan\theta = \tan 38° = 0.78$

b Let F be the required force up the plane. The net force up the plane is $F - mg\sin 49° - f_d$, since the force of friction now opposes F.

We have that:

$$f_d = \mu_s R = \mu_s mg\cos 49°$$

Therefore:

$$F - mg\sin 49° - \mu_s mg\cos 49° = ma$$

$$F = ma + mg\sin 49° + \mu_s mg\cos 49°$$

Substituting values:

$$F = 2.5 \times 1.2 + 2.5 \times 9.8 \times \sin 49° + 0.26 \times 2.5 \times 9.8\cos 49°$$

$$F = 25.67 \approx 26\,\text{N}$$

Exam tip

Notice that for a block on a frictionless inclined plane the net force down the plane is $mg\sin\theta$, leading to an acceleration of $g\sin\theta$, independent of the mass.

Nature of science

Physics and mathematics

In formulating his laws of motion, published in 1687 in *Philosophiæ Naturalis Principia Mathematica*, Newton used mathematics to show how the work of earlier scientists could be applied to forces and motion in the real world. Newton's second law (for particle of constant mass) is written as $F = ma$. In this form, this equation does not seem particularly powerful. However, using calculus, Newton showed that acceleration is given by:

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

The second law then becomes:

$$\frac{d^2x}{dt^2} - \frac{F}{m} = 0$$

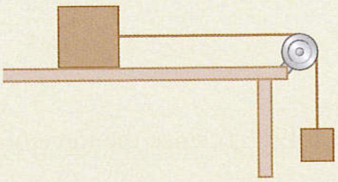
This is a differential equation that can be solved to give the actual path that the particle will move on under the action of the force. Newton showed that if the force depends on position as $F \propto \frac{1}{x^2}$, then the motion has to be along a conic section (ellipse, circle, etc.).

Newton used a flash of inspiration, triggered by observing an apple falling from a tree, to relate the motion of planets to that of the apple, leading to his law of gravitation (which you will meet in Topic **6**).

Test yourself

Equilibrium

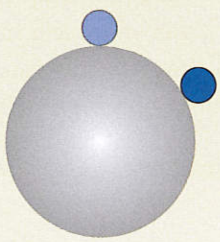
34 A block rests on a rough table and is connected by a string that goes over a pulley to a second hanging block, as shown in the diagram. Draw the forces on each body.



35 A bead rolls on the surface of a sphere, having started from the top, as shown in the diagram. On a copy of the diagram, draw the forces on the bead:

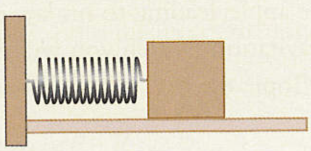
a at the top

b at the point where it is about to leave the surface of the sphere.



36 Look at the diagram. State in which case the tension in the string is largest.

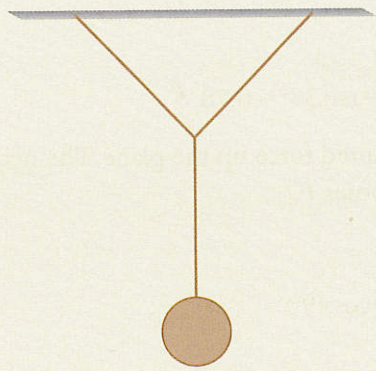
37 A spring is compressed by a certain distance and a mass is attached to its right end, as shown in the diagram. The mass rests on a rough table. On a copy of the diagram, draw the forces acting on the mass.



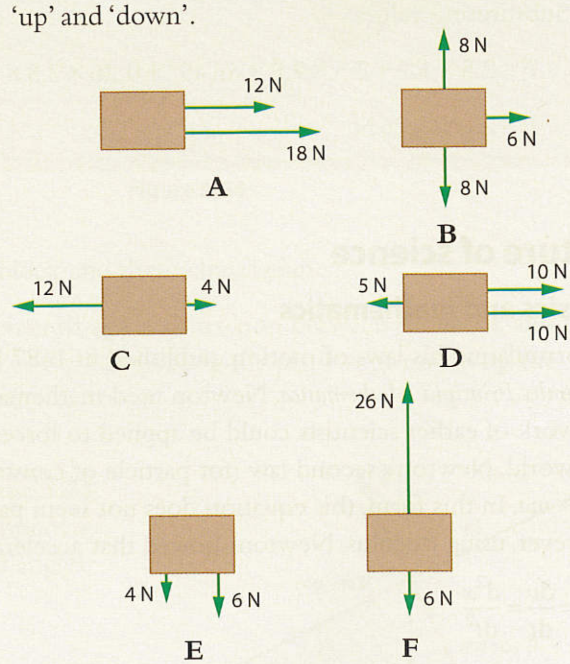
38 A mass hangs attached to three strings, as shown in the diagram. On a copy of the diagram, draw the forces on:

a the hanging mass

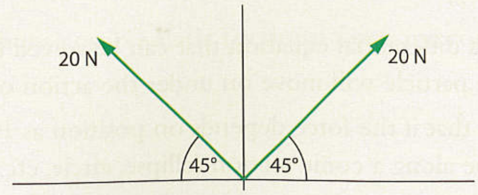
b the point where the strings join.



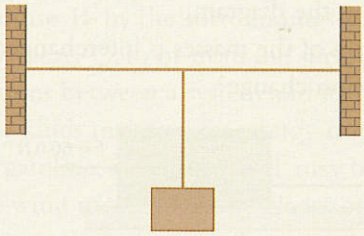
39 Find the net force on each of the bodies shown in the diagrams. The only forces acting are the ones shown. Indicate direction by 'right', 'left', 'up' and 'down'.



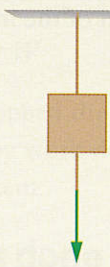
40 Find the magnitude and direction of the net force in the diagram.



41 Explain why is it impossible for a mass to hang attached to two horizontal strings as shown in the diagram.



42 A mass is hanging from a string that is attached to the ceiling. A second piece of string (identical to the first) hangs from the lower end of the mass (see diagram).

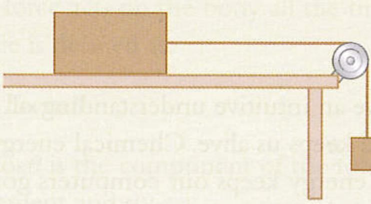


State and explain which string will break if:

a the bottom string is slowly pulled with ever increasing force

b the bottom string is very abruptly pulled down.

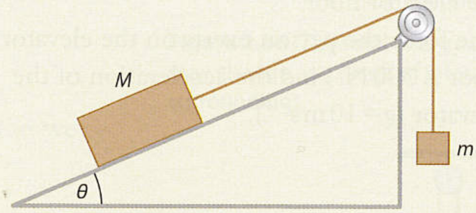
43 A mass of 2.00 kg rests on a rough horizontal table. The coefficient of static friction between the block and the table is 0.60. The block is attached to a hanging mass by a string that goes over a smooth pulley, as shown in the diagram. Determine the largest mass that can hang in this way without forcing the block to slide.



44 A girl tries to lift a suitcase of weight 220 N by pulling upwards on it with a force of 140 N. The suitcase does not move. Calculate the reaction force from the floor on the suitcase.

45 A block of mass 15.0 kg rests on a horizontal table. A force of 50.0 N is applied vertically downward on the block. Calculate the force that the block exerts on the table.

46 A block of mass M is connected with a string to a smaller block of mass m. The big block is resting on a smooth inclined plane as shown in the diagram. Determine the angle of the plane in terms of M and m in order to have equilibrium.



Accelerated motion

47 Describe under what circumstances a constant force would result in **a** an increasing and **b** a decreasing acceleration on a body.

48 A car of mass 1400 kg is on a muddy road. If the force from the engine pushing the car forward exceeds 600 N, the wheels slip (i.e. they rotate without rolling). Estimate the car's maximum acceleration on this road.

49 A man of mass m stands in an elevator.

a Find the reaction force from the elevator floor on the man when:

i the elevator is standing still

ii the elevator moves up at constant speed v

iii the elevator accelerates down with acceleration a

iv the elevator accelerates down with acceleration $a = g$.

b What happens when $a > g$?

50 Get in an elevator and stretch out your arm holding your heavy physics book. Press the button to go up. Describe and explain what is happening to your stretched arm. Repeat as the elevator comes to a stop at the top floor. What happens when you press the button to go down and what happens when the elevator again stops? Explain your observations carefully using the second law of motion.

51 The diagram shows a person in an elevator pulling on a rope that goes over a pulley and is attached to the top of the elevator. The mass of the elevator is 30.0 kg and that of the person is 70 kg.

a On a copy of the diagram, draw the forces on the person.

b Draw the forces on the elevator.

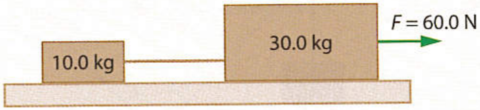
c The elevator accelerates upwards at $0.50\,\mathrm{m\,s^{-2}}$. Find the reaction force on the person from the elevator floor.

d The force the person exerts on the elevator floor is 300 N. Find the acceleration of the elevator ($g = 10\,\mathrm{m\,s^{-2}}$).

52 A massless string has the same tension throughout its length. Suggest why.

53 **a** Calculate the tension in the string joining the two masses in the diagram.

b If the position of the masses is interchanged, will the tension change?



54 A mass of 3.0 kg is acted upon by three forces of 4.0 N, 6.0 N and 9.0 N and is in equilibrium. Convince yourself that these forces can indeed be in equilibrium. The 9.0 N force is suddenly removed. Determine the acceleration of the mass.