Learning objectives

- Understand the concepts of kinetic, gravitational potential and elastic potential energy.
- Understand work done as energy transferred.
- Understand power as the rate of energy transfer.
- Understand and apply the principle of energy conservation.
- Calculate the efficiency in energy transfers.

2.3 Work, energy and power

This section deals with energy, one of the most basic concepts in physics. We introduce the principle of energy conservation and learn how to apply it to various situations. We define kinetic and potential energy, work done and power developed.

Energy

Energy is a concept that we all have an intuitive understanding of. Chemical energy derived from food keeps us alive. Chemical energy from gasoline powers our cars. Electrical energy keeps our computers going. Nuclear fusion energy produces light and heat in the Sun that sustains life on Earth. And so on. Very many experiments, from the subatomic to the cosmic scale, appear to be consistent with the principle of **conservation of energy** that states that energy is not created or destroyed but is only transformed from one form into another. This means that any change in the energy of a system must be accompanied by a change in the energy of the surroundings of the system such that:

$$\Delta E_{\text{system}} + \Delta E_{\text{surroundings}} = 0$$

In other words, if the system's energy increases, the energy of the surroundings must decrease by the same amount and vice-versa.

The energy of the system may change as a result of **interactions** with its surroundings (Figure **2.56**). These interactions mainly involve **work done** W by the surroundings and/or the **transfer of thermal energy** (heat) Q, to or from the surroundings. But there are many other interactions between a system and its surroundings. For example, waves of many kinds may transfer energy to the system (the Sun heats the Earth); gasoline, a chemical fuel, may be added to the system, increasing its energy; wind incident on the blades of a windmill will generate electrical energy as a generator is made to turn, etc. So:

$$\Delta E_{\text{system}} = W + Q + \text{other transfers}$$

But in this section we will deal with $Q = 0$ and no other transfers so we must understand and use the relation:

$$\Delta E = W$$

(we dropped the subscript in E_{system}). To do so, we need to define what we mean by work done and what exactly we mean by E, the total energy of the system.

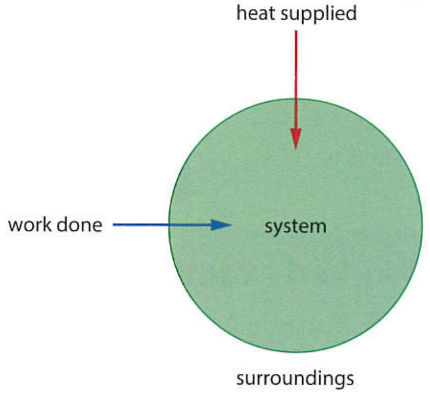


Figure 2.56 The total energy of a system may change as a result of interactions with its surroundings.

Work done by a force

We first consider the definition of **work done** by a constant force for motion in a straight line. By constant force we mean a force that is constant in magnitude as well as in direction. Figure **2.57** shows a block that is displaced along a straight line. The distance travelled by the body is s. The force makes an angle θ with the displacement.

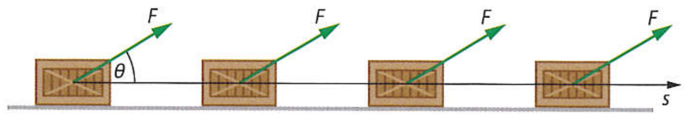


Figure 2.57 A force moving its point of application performs work.

The force acts on the body all the time as it moves. The work done by the force is defined as:

$$W = Fs\cos\theta$$

But $F\cos\theta$ is the component of the force in the direction of the displacement and so:

> The work done by a force is the product of the force in the direction of the displacement times the distance travelled.

(Equivalently, since $s\cos\theta$ is the distance travelled in the direction of the force, work may also be defined as the product of the force time, the distance travelled in the direction of the force.)

The cosine here can be positive, negative or zero; thus work can be positive, negative or zero. We will see what that means shortly.

The unit of work is the joule. One joule is the work done by a force of 1 N when it moves a body a distance of 1 m in the direction of the force. $1\,\text{J} = 1\,\text{N m}$.

Worked examples

2.27 A mass is being pulled along a level road by a rope attached to it in such a way that the rope makes an angle of 34° with the horizontal. The force in the rope is 24 N. Calculate the work done by this force in moving the mass a distance of 8.0 m along the level road.

We just have to apply the formula for work done:

$$W = Fs\cos\theta$$

Substituting the values from the question:

$$W = 24 \times 8.0 \times \cos 34°$$

$$W = 160\,\text{J}$$

2.28 A car with its engine off moves on a horizontal level road. A constant force of 620 N opposes the motion of the car. The car comes to rest after 84 m. Calculate the work done on the car by the opposing force.

We again apply the formula for work done, but now we have to realise that $\theta = 180°$. So:

$$W = 620 \times 84 \times \cos 180°$$

$$W = -52\,\text{kJ}$$

2.29 You stand on roller skates facing a wall. You push against the wall and you move away. Discuss whether the force exerted by the wall on you performed any work.

No work was done because there is no displacement. You moved but the point where the force is applied never moved.

Varying force and curved path

You will meet situations where the force is not constant in magnitude or direction and the path is not a straight line. To find the work done we must break up the curved path into very many small straight segments in a way that approximates the curved path (Figure **2.58**). Think of these segments as the dashes that make up the curve when it is drawn as a dashed line. The large arrowed segments at the bottom of Figure **2.58** show this more clearly. The total work done is the sum of the work done on each segment of the path.

We assume that along each segment the force is constant. The work done on the kth segment is just $F_k s_k \cos\theta_k$. So the work done on all the segments is found by adding up the work done on individual segments, i.e.

$$W = \sum_{k=1} F_k s_k \cos\theta_k$$

Do not be too worried about this formula. You will not be asked to use it, but it can help you to understand one very special and important case: the work done in circular motion. We will learn in Topic **6** that in circular motion there must be a force directed towards the centre of the circle. This is called the **centripetal force**.

Figure **2.59** shows the forces pointing towards the centre of the circular path. When we break the circular path into straight segments the angle between the force and the segment is always a right angle. This means that work done along each segment is zero because $\cos 90° = 0$. So for circular motion the total work done by the centripetal force is zero.

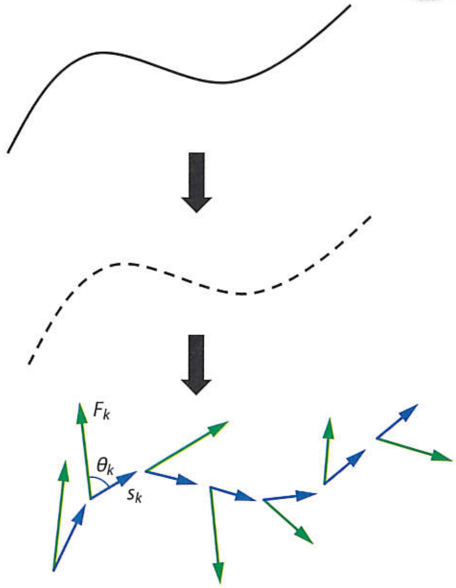


Figure 2.58 The curved path followed by a particle is shown as a dashed line, and then as larger segments, s_k. The green arrows show the varying size and direction of the force acting on the particle as it moves.

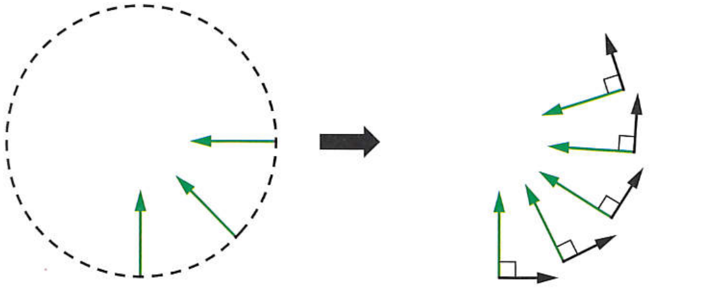


Figure 2.59 The work done by the centripetal force is zero.

In practice, when the force varies in magnitude but is constant in direction, we will be given a graph of how the force varies with distance travelled. The work done can be found from the area under the graph. For the motion shown in Figure **2.60**, the work done in moving a distance of 4.0 m is given by the area of the shaded trapezoid:

$$W = \frac{2.0 + 10}{2} \times 4.0 = 24\,\text{J}$$

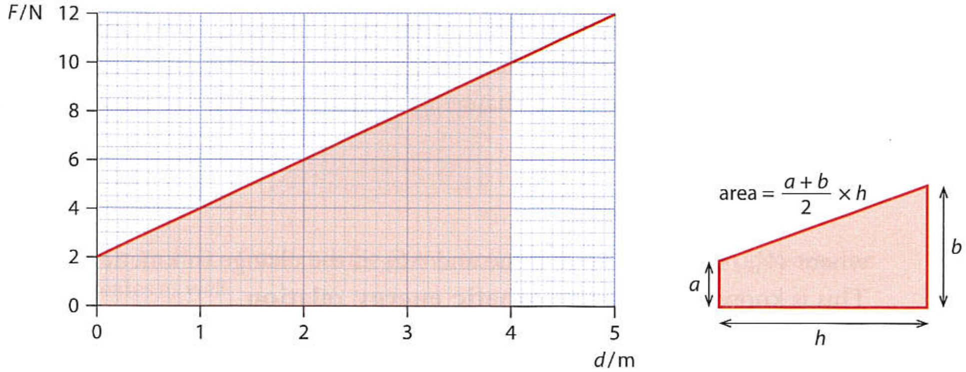


Figure 2.60 The work done is the area under the graph. The area of a trapezoid is half the sum of the parallel sides multiplied by the perpendicular distance between them.

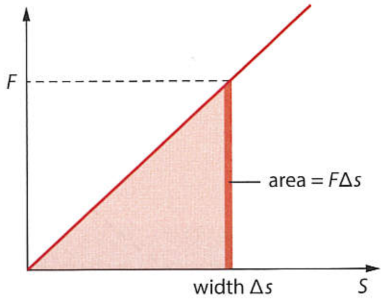


Figure 2.61 The area under the graph is the sum of all the rectangles $F\Delta s$.

The work done by a force is the area under the graph that shows the variation of the magnitude of the force with distance travelled.

How do we know that the area is the work done? For a varying force, consider a very small distance Δs (Figure **2.61**). Because Δs is so small we may assume that the force does not vary during this distance. The work done is then $F\Delta s$ and is the area of the rectangle shown. For the total work we have to add the area of many rectangles under the curve. The sum is the area under the curve.

Work done by a force on a particle

Imagine a net force F that acts on a particle of mass m. The force produces an acceleration a given by:

$$a = \frac{F}{m}$$

Let the initial speed of the particle be u. Because we have acceleration, the speed will change. Let the speed be v after travelling a distance s. We know from kinematics that:

$$v^2 = u^2 + 2as$$

Substituting for the acceleration, this becomes:

$$v^2 = u^2 + 2\frac{F}{m}s$$

We can rewrite this as:

$$Fs = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2$$

We interpret this as follows: Fs is the work done on the particle by the net force. The quantity $\frac{1}{2} \times \text{mass} \times \text{speed}^2$ is the energy the particle has due to its motion, called kinetic energy. For speed v, **kinetic energy** E_K is defined as:

$$E_K = \tfrac{1}{2}mv^2$$

In our example, the initial kinetic energy of the particle is $\frac{1}{2}mu^2$ and the kinetic energy after travelling distance s is $\frac{1}{2}mv^2$. The result says that the work done has gone into the change in the kinetic energy of the particle.

We can write this as:

$$W_{net} = \Delta E_K$$

where W_{net} is the net work done and ΔE_K is the change in kinetic energy. This is known as the **work–kinetic energy relation**.

We can think of the work done as energy transferred. In this example, the work done has transferred energy to the particle by increasing its kinetic energy.

Worked example

2.30 A block of mass 2.5 kg slides on a rough horizontal surface. The initial speed of the block is $8.6\,\text{m s}^{-1}$. It is brought to rest after travelling a distance of 16 m. Determine the magnitude of the frictional force.

We will use the work–kinetic energy relation, $W_{net} = \Delta E_K$.

The only force doing work is the frictional force, f, which acts in the opposite direction to the motion.

$W_{net} = f \times 16 \times (-1)$ — The angle between the force and the direction of motion is 180°, so we need to multiply by cos 180°, which is −1.

The change in kinetic energy is:

$$\Delta E_K = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2 = -92.45\,\text{J}$$

So: $-16f = -92.45$

$f = 5.8\,\text{N}$

The magnitude of the frictional force is 5.8 N.

Work done in stretching a spring

Consider a horizontal spring whose left end is attached to a vertical wall. If we apply a force F to the other end we will stretch the spring by some amount, x. Experiments show that the force F and the extension x are directly proportional to each other, i.e. $F = kx$ (this is known as **Hooke's law**). How much work does the stretching force F do in stretching the spring from its natural length (i.e. from zero extension) to a length where the extension is x_1, as shown in Figure **2.62**.

Since the force F and the extension x are directly proportional, the graph of force versus extension is a straight line through the origin and work done is the area under the curve (Figure **2.63**).

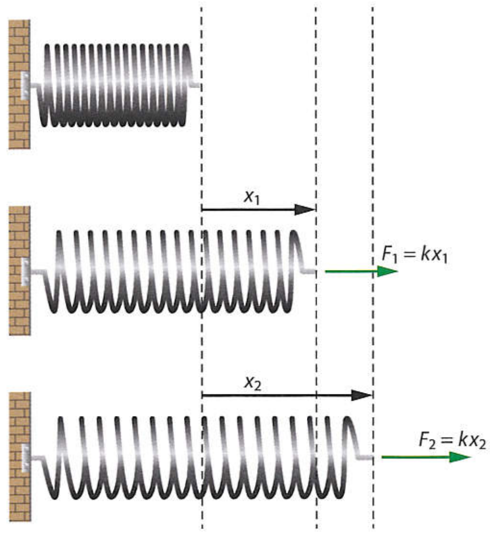


Figure 2.62 Stretching a spring requires work to be done.

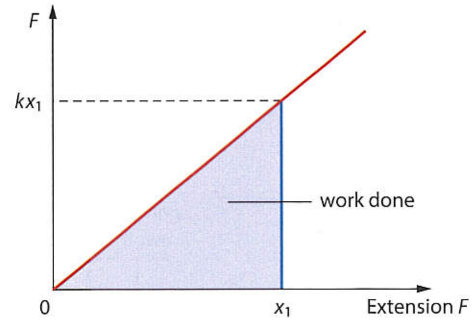


Figure 2.63 The force F stretches the spring. Notice that as the extension increases the force increases as well.

To find the work done in extending the spring from its natural length ($x=0$) to extension x_1, we need to calculate the area of the triangle of base x_1 and height kx_1. Thus:

$$\text{area} = \tfrac{1}{2}kx_1 \times x_1$$

$$\text{area} = \tfrac{1}{2}kx_1{}^2$$

The work to extend a spring from its natural length by an amount x_1 is thus:

$$W = \tfrac{1}{2}kx_1{}^2$$

It follows that the work done when extending a spring from an extension x_1 to an extension x_2 (so $x_2 > x_1$) is:

$$W = \tfrac{1}{2}k(x_2{}^2 - x_1{}^2)$$

The work done by the force extending the spring goes into elastic potential energy stored in the spring. The elastic potential energy of a spring whose extension is x is $E_{el} = \tfrac{1}{2}kx^2$.

Exam tip
In discussing work done it is always important to keep a clear picture of the force whose work we are calculating.

Worked example

2.31 A mass of 8.4 kg rests on top of a vertical spring whose base is attached to the floor. The spring compresses by 5.2 cm.

a Calculate the spring constant of the spring.

b Determine the energy stored in the spring.

a The mass is at equilibrium so $mg = kx$. So:

$$k = \frac{mg}{x}$$

$$k = \frac{8.4 \times 9.8}{5.2 \times 10^{-2}}$$

$$k = 1583 \approx 1600\,\text{N}\,\text{m}^{-1}$$

b The stored energy E_{el} is:

$$E_{el} = \tfrac{1}{2}kx^2$$

$$E_{el} = \tfrac{1}{2} \times 1583 \times (5.2 \times 10^{-2})^2$$

$$E_{el} = 2.1\,\text{J}$$

Work done by gravity

We will now concentrate on the work done by a very special force, namely the weight of a body. Remember that weight is mass times acceleration of free fall and is directed vertically down. Thus, if a body is displaced horizontally, the work done by mg is zero. In this case the angle between the force and the direction of motion is 90° (Figure **2.64**), so:

$$W = mgs\cos 90° = 0$$

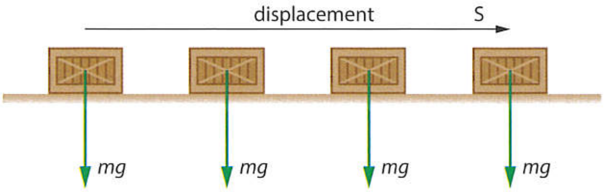


Figure 2.64 The force of gravity is normal to this horizontal displacement, so no work is being done.

Exam tip

When a body is displaced such that its final position is at the same vertical height as the original position, the work done by the weight is zero.

We are not implying that it is the weight that is forcing the body to move along the table. We are calculating the work done by a particular force (the weight) if the body (somehow) moves in a particular way.

If the body falls a vertical distance h, then the work done by the weight is $+mgh$. The force of gravity is parallel to the displacement, as in Figure **2.65a**.

If the body moves vertically upwards to a height h from the launch point, then the work done by the weight is $-mgh$ since now the angle between direction of force (vertically down) and displacement (vertically up) is 180°. The force of gravity is parallel to the displacement but opposite in direction, as in Figure **2.65b**.

Suppose now that instead of just letting the body fall or throwing it upwards, we use a rope to either lower it or raise it, at constant speed, by a height h (Figure **2.66**). The work done by the weight is the same as before, so nothing changes. But we now ask about the work done by the force F that lowers or raises the body. Since F is equal and opposite to the weight, the work done by F is $-mgh$ as the body is lowered and $+mgh$ as it is being raised.

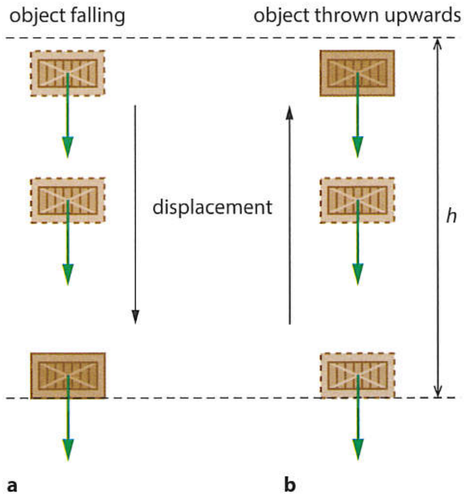


Figure 2.65 The force of gravity (green arrows) is parallel to the displacement in **a** and opposite in **b**.

You should be able to see how this is similar to the work done by the stretching and tension forces in a spring.

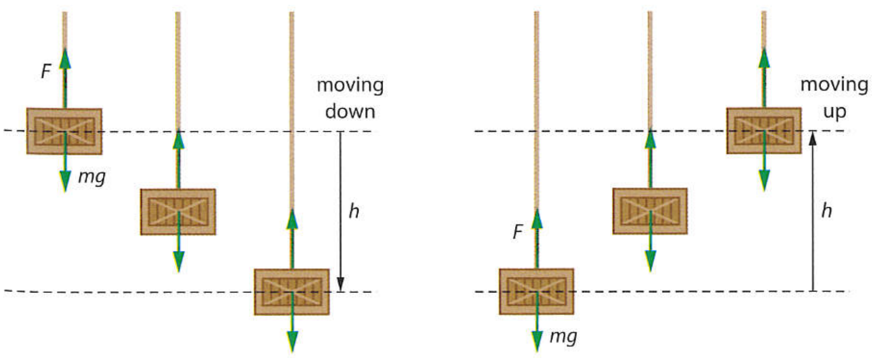


Figure 2.66 Lowering and raising an object at constant speed using a rope.

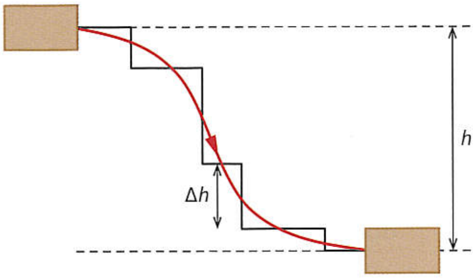


Figure 2.67 The work done by gravity is independent of the path followed.

Consider now the case where a body moves along some arbitrary path, as shown by the red line in Figure **2.67**. The work done by the weight of the body as the body descends along the curve is still mgh. You can prove this amazing result easily by approximating the curved path with a 'staircase' of vertical and horizontal steps. Along the horizontal steps the work done is zero, $\cos 90° = 0$. Along the vertical steps the work is $mg\Delta h$, where Δh is the step height. Adding up all the vertical steps gives mgh. This means that:

The work done by gravity is independent of the path followed and depends only on the vertical distance separating the initial and final positions.

The independence of the work done on the path followed is a property of a class of forces (of which weight is a prominent member) called **conservative forces**.

Mechanical energy

In the previous two sections we discussed the work done when a body is moved when attached to a spring and in a gravitational field. We derived two main results.

In the case of the spring, we showed that the work done by the stretching force in extending the spring a distance x away from the natural length of the spring is $W = \frac{1}{2}x^2$.

In the case of motion within a gravitational field the work done by the force moving the body, is $W = mgh$ to raise the body a height h from its initial position.

We use these results to define two different kinds of **potential energy**, E_P.

For the mass–spring system we define the **elastic potential energy** to be the work done by the pulling force in stretching the spring by an amount x, that is:

$$E_P = \frac{1}{2}kx^2$$

For the Earth–mass system we define the **gravitational potential energy** to be the work done by the moving force in placing a body a height h above its initial position, that is:

$$E_P = mgh$$

Notice that potential energy is the property of a system, not of an individual particle.

So we are now in a position to go back to the first part of Subtopic **2.3** and answer some of the questions posed there. We said that:

$$\Delta E = W + Q$$

Exam tip
Potential energy is the energy of a system due to its position or shape and represents the work done by an external agent in bringing the system to that position or shape.

Exam tip
Notice that in the data booklet the formula uses Δx in place of our x.

If the system is in contact with surroundings at a different temperature there will be a transfer of heat, Q. If there is no contact and no temperature difference, then $Q=0$.

If no work is done on the system from outside, then $W=0$. When $Q+W=0$, the system is called **isolated** and in that case $\Delta E=0$. The total energy of the system does not change. We have **conservation of the total energy** of the system.

What does the total energy E consist of? It includes chemical energy, **internal energy** (due to the translational, rotational energy and vibrational energy of the molecules of the substance), nuclear energy, kinetic energy, elastic potential energy, gravitational potential energy and any other form of potential energy such as electrical potential energy.

But in this section, dealing with mechanics, the total energy E will be just the sum of the kinetic, the elastic and the gravitational potential energies.

So for a single particle of mass m, the energy is:

$$E=\tfrac{1}{2}mv^2+mgh+\tfrac{1}{2}kx^2$$

This is also called the **total mechanical energy** of the system consisting of the particle, the spring and the Earth. W stands for work done by forces outside the system. So this does not include work due to spring tension forces or the weight since the work of these forces is already included as potential energy in E.

Exam tip

You must make sure that you do not confuse the work–kinetic energy relation $W_{net}=\Delta E_K$ with $\Delta E=W$. The work–kinetic energy relation relates the net work on a system to the change in the system's kinetic energy. The other relates the work done by outside forces to the change of the total energy.

Worked examples

2.32 You hold a ball of mass 0.25 kg in your hand and throw it so that it leaves your hand with a speed of $12\,\mathrm{m\,s^{-1}}$. Calculate the work done by your hand on the ball.

The question asks for work done but here we do not know the forces that acted on the ball nor the distance by which we moved it before releasing it. But using $\Delta E=W$, we find:

$$W=\tfrac{1}{2}mv^2$$

$$W=\tfrac{1}{2}\times0.25\times12^2=36\,\mathrm{J}$$

Notice that here we have no springs and we may take $h=0$.

2.33 Suppose that in the previous example your hand moved a distance of 0.90 m in throwing the ball. Estimate the average net force that acted on the ball.

The work done was 36 J and so $Fs=36\,\mathrm{J}$ with $s=0.90\,\mathrm{m}$. This gives $F=40\,\mathrm{N}$.

2.34 A body of mass 4.2 kg with initial speed $5.6\,\text{m}\,\text{s}^{-1}$ begins to move up an incline, as shown in Figure **2.68**.

Figure 2.68

The body will be momentarily brought to rest after colliding with a spring of spring constant $220\,\text{N}\,\text{m}^{-1}$. The body stops a vertical distance 0.85 m above its initial position. Determine the amount by which the spring has been compressed. There are no frictional forces.

There are no external forces doing work and so $W = 0$. The system is isolated and we have conservation of total energy.

Initially we have just kinetic energy, so:

$$E_{\text{initial}} = \tfrac{1}{2}mv^2 + mgh + \tfrac{1}{2}kx^2 = \tfrac{1}{2} \times 4.2 \times 5.6^2 + 0 + 0 = 65.856\,\text{J}$$

When the body stops we have:

$$E_{\text{initial}} = \tfrac{1}{2}mv^2 + mgh + \tfrac{1}{2}kx^2 = 0 + 4.2 \times 9.8 \times 0.85 + \tfrac{1}{2} \times 220 \times x^2 = 34.99 + 110x^2$$

Thus, equating E_{initial} to E_{final} we find:

$$\begin{aligned} 34.99 + 110x^2 &= 65.856 \\ 110x^2 &= 30.866 \\ x^2 &= 0.2806 \\ x &= 0.53\,\text{m} \end{aligned}$$

2.35 We repeat the previous example question but now there is constant frictional force opposing the motion along the uphill part of the path. The length of this path is 1.2 m and the frictional force is 15 N.

We have $\Delta E = W$. The work done is:

$$Fs\cos\theta = 15 \times 1.2 \times (-1) = -18\,\text{J}$$

As in the previous example, we have:

$$\begin{aligned} E_{\text{initial}} &= 65.856\,\text{J} \\ E_{\text{final}} &= 34.99 + 110x^2 \end{aligned}$$

leading to:

$$\begin{aligned} 110x^2 &= 12.866 \\ x^2 &= \frac{12.866}{110} \\ x &= 0.34\,\text{m} \end{aligned}$$

The 'work done by friction' of $-18\,\text{J}$ is energy that is dissipated as thermal energy inside the body *and* its surroundings. It is in general very difficult to estimate how much of this thermal energy stays within the body and how much goes into the surroundings.

2.36 A mass of 5.00 kg moving with an initial velocity of $2.0\,\mathrm{m\,s^{-1}}$ is acted upon by a force 55 N in the direction of the velocity. The motion is opposed by a frictional force. After travelling a distance of 12 m the velocity of the body becomes $15\,\mathrm{m\,s^{-1}}$. Determine the magnitude of the frictional force.

Here $Q=0$ so that $\Delta E = W$.

The change in total energy ΔE is the change in kinetic energy (we have no springs and no change of height):

$$\Delta E = \tfrac{1}{2}\times 5.00\times 15^2 - \tfrac{1}{2}\times 5.00\times 2.0^2 = 552.5\,\mathrm{J}$$

Let the frictional force be f. The work done on the mass is $(55-f)\times 12$, and so:

$$(55-f)\times 12 = 552.5$$

$$55-f=\frac{552.5}{12}$$

$$55-f=46.0$$

$$f=9.0\,\mathrm{N}$$

The 'work done by friction' of $-9.0\times 12 = -108\,\mathrm{J}$ is energy that is dissipated as thermal energy inside the body *and* its surroundings.

2.37 A mass m hangs from two strings attached to the ceiling such that they make the same angle with the vertical (as shown in Figure **2.69**). The strings are shortened very slowly so that the mass is raised a distance Δh above its original position. Determine the work done by the tension in each string as the mass is raised.

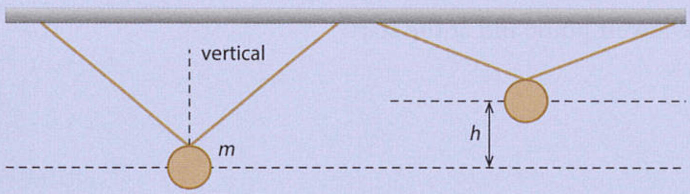


Figure 2.69

The net work done is zero since the net force on the mass is zero. The work done by gravity is $-mg\Delta h$ and thus the work done by the two equal tension forces is $+mg\Delta h$. The work done by each is thus $\frac{mg\Delta h}{2}$.

2.38 A pendulum of length 1.0 m is released from rest with the string at an angle of 10° to the vertical. Find the speed of the mass on the end of the pendulum when it passes through its lowest position.

Let us take as the reference level the lowest point of the pendulum (Figure **2.70**). The total energy at that point is just kinetic, $E_K = \frac{1}{2}mv^2$, where v is the unknown speed.

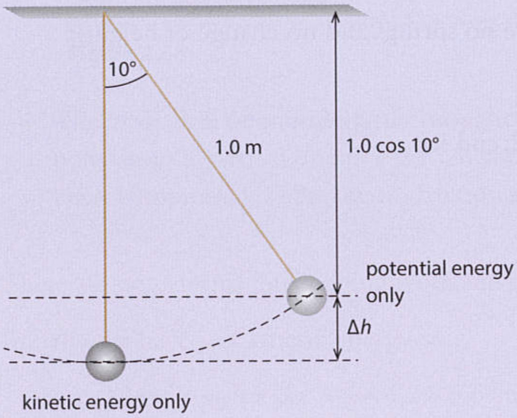


Figure 2.70

At the initial point, the total energy is just potential, $E_P = mg\Delta h$, where Δh is the vertical difference in height between the two positions. From the diagram:

$$\Delta h = 1.00 - 1.00 \cos 10°$$

$$\Delta h = 0.015\,\text{m}$$

Equating the expressions for the total energy at the lowest point and at the start:

$$\frac{1}{2}mv^2 = mg\Delta h$$

$$v = \sqrt{2g\Delta h}$$

$$v = 0.55\,\text{m}\,\text{s}^{-1}$$

Note how the mass has dropped out of the problem. (At positions other than the two shown, the mass has both kinetic and potential energy.)

2.39 Determine the minimum speed of the mass in Figure **2.71** at the initial point such that the mass makes it over the barrier of height h.

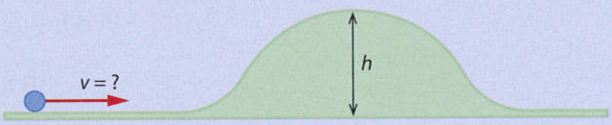


Figure 2.71

To make it over the barrier the mass must be able to reach the highest point. Any speed it has at the top will mean it can carry on to the other side. Therefore, at the very least, we must be able to get the ball to the highest point with zero speed.

With zero speed at the top, the total energy at the top of the barrier is $E = mgh$.

The total energy at the starting position is $\frac{1}{2}mv^2$.

Equating the initial and final energy:

$$\frac{1}{2}mv^2 = mgh$$

$$\Rightarrow \ v = \sqrt{2gh}$$

Thus, the initial speed must be bigger than $v = \sqrt{2gh}$.

Note that if the initial speed u of the mass is larger than $v = \sqrt{2gh}$, then when the mass makes it to the original level on the other side of the barrier, its speed will be the same as the starting speed u.

2.40 A ball rolls off a 1.0 m high table with a speed of $4.0\,\text{m s}^{-1}$, as shown in Figure **2.72**. Calculate the speed as the ball strikes the floor.

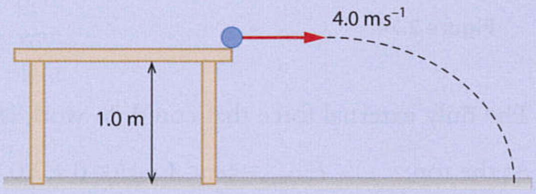


Figure 2.72

The total energy of the mass is conserved. As it leaves the table with speed u it has total energy given by $E_{\text{initial}} = \frac{1}{2}mu^2 + mgh$ and as it lands with speed v the total energy is $E_{\text{final}} = \frac{1}{2}mv^2$ (v is the speed we are looking for).

Equating the two energies gives:

$$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mgh$$

$$\Rightarrow \ v^2 = u^2 + 2gh$$

$$v^2 = 16 + 20 = 36$$

$$\Rightarrow \ v = 6.0\,\text{m s}^{-1}$$

2.41 Two identical balls are launched from a table with the same speed u (Figure **2.73**). One ball is thrown vertically up and the other vertically down. The height of the table from the floor is h. Predict which of the two balls will hit the floor with the greater speed.

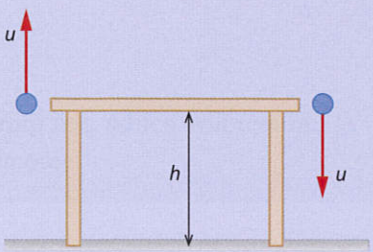


Figure 2.73

At launch both balls have the same kinetic energy and the same potential energy. When they hit the floor their energy will be only kinetic. Hence the speeds will be identical and equal to v, where:

$$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mgh$$

$$\Rightarrow \ v^2 = u^2 + 2gh$$

$$\Rightarrow \ v = \sqrt{u^2 + 2gh}$$

2.42 A body of mass 2.0 kg (initially at rest) slides down a curved path of total length 22 m, as shown in Figure **2.74**. The body starts from a vertical height of 5.0 m from the bottom. When it reaches the bottom, its speed is measured and found to equal $6.0\,\mathrm{m\,s^{-1}}$.

a Show that there is a force resisting the motion.

b Assuming the force to have constant magnitude, determine the magnitude of the force.

Figure 2.74

a The only external force that could do work is a frictional force.

At the top: $E_{\text{initial}} = \frac{1}{2}mv^2 + mgh = 0 + 2.0 \times 9.8 \times 5.0 = 98\,\mathrm{J}$

At the bottom: $E_{\text{final}} = \frac{1}{2}mv^2 + mgh = \frac{1}{2} \times 2.0 \times 6.0^2 + 0 = 36\,\mathrm{J}$

The total energy has reduced, which shows the presence of a frictional force resisting the motion.

b From $\Delta E = W$ we deduce that $W = -62\,\mathrm{J}$. This is the work done by the frictional force, magnitude f.

The force acts in the opposite direction to the motion, so:

$$fs \times (-1) = -62\,\mathrm{J}$$

$$\Rightarrow f = \frac{62}{22}$$

$$f = 2.8\,\mathrm{N}$$

Power

When a machine performs work, it is important to know not only how much work is being done but also how much work is performed within a given time interval. A cyclist will perform a lot of work in a lifetime of cycling, but the same work can be performed by a powerful car engine in a much shorter time. **Power** is the rate at which work is being performed or the rate at which energy is being transferred.

When a quantity of work ΔW is performed within a time interval Δt the power developed is given by the ratio:

$$P = \frac{\Delta W}{\Delta t}$$

is called the power developed. Its unit is joule per second and this is given the name watt (W): $1\,\mathrm{W} = 1\,\mathrm{J\,s^{-1}}$.

Consider a constant force F, which acts on a body of mass m. The force does an amount of work $F\Delta x$ in moving the body a small distance Δx along its direction. If this work is performed in time Δt, then:

$$P = \frac{\Delta W}{\Delta t}$$

$$P = F\frac{\Delta x}{\Delta t}$$

$$P = Fv$$

where v is the instantaneous speed of the body. This is the power produced in making the body move at speed v. As the speed increases, the power necessarily increases as well.

Consider an aeroplane moving at constant speed on a straight-line path. If the power produced by its engines is P, and the force pushing it forward is F, then P, F and v are related by the equation above. But since the plane moves with no acceleration, the total force of air resistance must equal F. Hence the force of air resistance can be found simply from the power of the plane's engines and the constant speed with which it coasts.

Worked example

2.43 Estimate the minimum power required to lift a mass of 50.0 kg up a vertical distance of 12 m in 5.0 s.

The work done in lifting the mass is mgh:

$$W = mgh = 50.0 \times 10 \times 12$$

$$W = 6.0 \times 10^3\,\text{J}$$

The power is therefore:

$$P = \frac{W}{\Delta t}$$

$$P = \frac{6.0 \times 10^3}{5.0} = 1200\,\text{W}$$

This is the minimum power required. In practice, the mass has to be accelerated from rest, which will require additional work and hence more power. There will also be frictional forces to overcome.

Efficiency

If a machine, such as an electric motor, is used to raise a load, electrical energy must be provided to the motor. This is the input energy to the motor. The motor uses some of this energy to do the useful work of raising the load. But some of the input energy is used to overcome frictional forces and therefore gets converted to thermal energy. So the ratio:

$$\frac{\text{useful energy out}}{\text{actual energy in}} \quad \text{or} \quad \frac{\text{useful power out}}{\text{actual power in}}$$

is less than one. We call this ratio the **efficiency** of the machine.

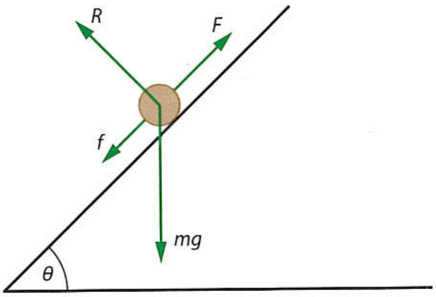


Figure 2.75 Forces on a body on an inclined plane: pulling force F, frictional force f, reaction R and weight mg.

Suppose that a body is being pulled up along a rough inclined plane with constant speed. The mass is 15 kg and the angle of the incline is 45°. There is a constant frictional force of 42 N opposing the motion.

The forces on the body are shown in Figure **2.75**. Since the body has no acceleration, we know that:

$$R = mg\cos\theta = 106.1\,\text{N}$$

$$F = mg\sin\theta + f = 106.1 + 42 = 148.1\,\text{N} \approx 150\,\text{N}$$

Let the force raise the mass a distance of 20 m along the plane. The work done by the force F is:

$$W = 148.1 \times 20$$

$$W = 2960\,\text{J} \approx 3.0 \times 10^3\,\text{J}$$

The force effectively raised the 15 kg a vertical height of 14.1 m and so increased the potential energy of the mass by $mgh = 2121$ J. The efficiency with which the force raised the mass is thus:

$$\text{efficiency} = \frac{2121}{2960}$$

$$\text{efficiency} = 0.72$$

Worked example

2.44 A 0.50 kg battery-operated toy train moves with constant velocity 0.30 m s^{-1} along a level track. The power of the motor in the train is 2.0 W and the total force opposing the motion of the train is 5.0 N.

a Determine the efficiency of the train's motor.

b Assuming the efficiency and the opposing force stay the same, calculate the speed of the train as it climbs an incline of 10.0° to the horizontal.

a The power delivered by the motor is 2.0 W. Since the speed is constant, the force developed by the motor is also 5.0 N.

The power used in moving the train is $Fv = 5.0 \times 0.30 = 1.5$ W.

Hence the efficiency is:

$$\frac{\text{total power out}}{\text{total power in}} = \frac{1.5\,\text{W}}{2.0\,\text{W}}$$

$$\frac{\text{total power out}}{\text{total power in}} = 0.75$$

The efficiency of the train's motor is 0.75 (or 75%).

b The component of the train's weight acting down the plane is $mg\sin\theta$ and the force opposing motion is 5.0 N. Since there is no acceleration (constant velocity), the net force F pushing the train up the incline is:

$$F = mg\sin\theta + 5.0$$

$$F = 0.50 \times 10 \times \sin 10° + 5.0$$

$$F = 5.89\,\text{N} \approx 5.9\,\text{N}$$

Thus:

$$\text{efficiency} = \frac{5.89 \times v}{2.0}$$

But from part **a** the efficiency is 0.75, so:

$$0.75 = \frac{5.89 \times v}{2.0}$$

$$\Rightarrow v = \frac{2.0 \times 0.75}{5.89}$$

$$v = 0.26\,\text{m s}^{-1}$$

Nature of science

The origin of conservation principles

Understanding of what energy is has evolved over time, with Einstein showing that there is a direct relationship between mass and energy in his famous equation $E = mc^2$. In this section we have seen how the principle of conservation of energy can be applied to different situations to predict and explain what will happen. Scientists have been able to use the theory to predict the outcome of previously unknown interactions in particle physics.

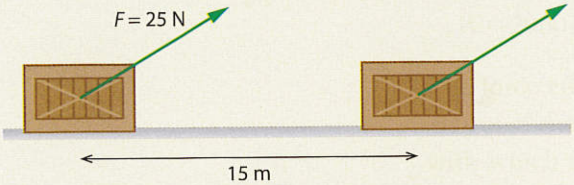
The principle of conservation of energy is perhaps the best known example of a conservation principle. But where does it come from? It turns out that all conservation principles are consequences of symmetry. In the case of energy, the symmetry is that of 'time translation invariance'. This means that when describing motion (or anything else) it does not matter when you started the stopwatch. So a block of mass 1 kg on a table 1 m above the floor will have a potential energy of 10 J according to both an observer who starts his stopwatch 'now' and another who started it 10 seconds ago. The principle of conservation of momentum, which is discussed in Subtopic **2.4**, is also the result of a symmetry. The symmetry this time is 'space translation invariance', which means that in measuring the position of events it does not matter where you place the origin of your ruler.

? Test yourself

55 A horizontal force of 24 N pulls a body a distance of 5.0 m along its direction. Calculate the work done by the force.

56 A block slides along a rough table and is brought to rest after travelling a distance of 2.4 m. A force of 3.2 N opposes the motion. Calculate the work done by the opposing force.

57 A block is pulled as shown in the diagram by a force making an angle of 20° to the horizontal. Find the work done by the pulling force when its point of application has moved 15 m.



58 A block of mass 2.0 kg and an initial speed of 5.4 m s^{-1} slides on a rough horizontal surface and is eventually brought to rest after travelling a distance of 4.0 m. Calculate the frictional force between the block and the surface.

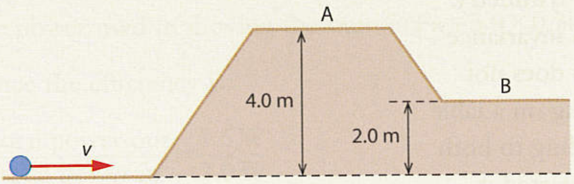
59 A spring of spring constant k = 200 N m^{-1} is slowly extended from an extension of 3.0 cm to an extension of 5.0 cm. Calculate the work done by the extending force.

60 Look at the diagram.

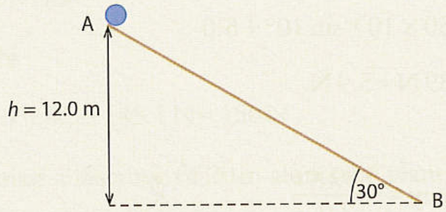
a **i** Calculate the minimum speed v the ball must have in order to make it to position **B**.

ii What speed will the mass have at **B**?

b Given that v = 12.0 m s^{-1}, calculate the speed at **A** and **B**.

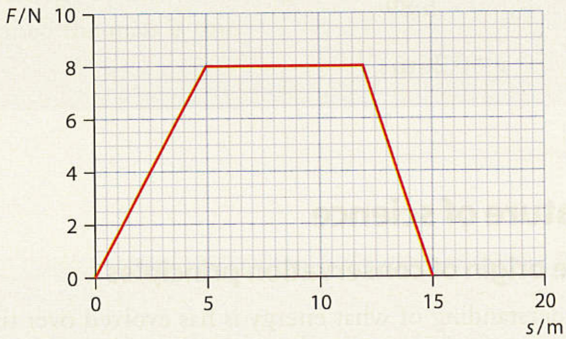


61 The speed of the 8.0 kg mass in position **A** in the diagram is 6.0 m s^{-1}. By the time it gets to **B** the speed is measured to be 12.0 m s^{-1}.



Estimate the frictional force opposing the motion. (The frictional force is acting along the plane.)

62 A force F acts on a body of mass m = 2.0 kg initially at rest. The graph shows how the force varies with distance travelled (along a straight line).



a Find the work done by this force.

b Calculate the final speed of the body.

63 A body of mass 12 kg is dropped vertically from rest from a height of 80 m.

a Ignoring any resistance forces during the motion of this body, draw graphs to represent the variation with distance fallen of:

i the potential energy

ii the kinetic energy.

b For the same motion draw graphs to represent the variation with time of:

i the potential energy

ii the kinetic energy.

c Describe qualitatively the effect of a constant resistance force on each of the four graphs you drew.

64 The engine of a car is developing a power of 90 kW when it is moving on a horizontal road at a constant speed of 100 km h^{-1}. Estimate the total horizontal force opposing the motion of the car.

65 The motor of an elevator develops power at a rate of 2500 W.

a Calculate the speed that a 1200 kg load is being raised at.

b In practice it is found that the load is lifted more slowly than indicated by your answer to **a**. Suggest reasons why this is so.

66 A load of 50 kg is raised a vertical distance of 15 m in 125 s by a motor.

a Estimate the power necessary for this.

b The power supplied by the motor is in fact 80 W. Calculate the efficiency of the motor.

c The same motor is now used to raise a load of 100 kg the same distance. The efficiency remains the same. Estimate how long this would take.

67 The top speed of a car whose engine is delivering 250 kW of power is 240 km h^{-1}. Calculate the value of the resistance force on the car when it is travelling at its top speed on a level road.

68 An elevator starts on the ground floor and stops on the 10th floor of a high-rise building. The elevator reaches a constant speed by the time it reaches the 1st floor and decelerates to rest between the 9th and 10th floors. Describe the energy transformations taking place between the 1st and 9th floors.

69 A mass m of 4.0 kg slides down a frictionless incline of $\theta = 30°$ to the horizontal. The mass starts from rest from a height of 20 m.

a Sketch a graph of the kinetic and potential energies of the mass as a function of time.

b Sketch a graph of the kinetic and potential energies of the mass as a function of distance travelled along the incline.

c On each graph, sketch the sum of the potential and kinetic energies.

70 A mass m is being pulled up an inclined plane of angle θ by a rope along the plane.

a Find is the tension in the rope if the mass moves up at constant speed v.

b Calculate is the work done by the tension when the mass moves up a distance of d m along the plane.

c Find is the work done by the weight of the mass.

d Find is the work done by the normal reaction force on the mass.

e What is the net work done on the mass?

71 A battery toy car of mass 0.250 kg is made to move up an inclined plane that makes an angle of 30° with the horizontal. The car starts from rest and its motor provides a constant acceleration of 4.0 m s^{-2} for 5.0 s. The motor is then turned off.

a Find the distance travelled in the first 5 s.

b Find the furthest the car gets on the inclined plane.

c Calculate when the car returns to its starting position.

d Sketch a graph of the velocity as a function of time.

e On the same axes, sketch a graph of the kinetic energy and potential energy of the car as a function of the distance travelled.

f State the periods in the car's motion in which its mechanical energy is conserved.

g Estimate the average power developed by the car's motor.

h Determine the maximum power developed by the motor.