

## 4.1 KINEMATICS OF SIMPLE HARMONIC MOTION

## 4.2 ENERGY CHANGES DURING SIMPLE HARMONIC MOTION

## 4.3 FORCED OSCILLATIONS AND RESONANCE

HW/Study Packet

SL/HL	
<b>Required:</b> READ Hamper pp 100-114 Tsokos, pp 195-209	<b>Supplemental:</b> DO Tsokos pp 209-215, #9,10,13,18,20,26 READ Cutnell and Johnson, pp 285-302 (HL)

### REMEMBER TO....

- ✓ *Work through all of the 'example problems' in the texts as you are reading them*
- ✓ *Refer to the **IB Physics Guide** for details on what you need to know about this topic*
- ✓ *Refer to the **Study Guides** for suggested exercises to do each night*
- ✓ *First try to do these problems using only what is provided to you from the **IB Data Booklet***
- ✓ *Refer to the solutions/key **ONLY** after you have attempted the problems to the best of your ability*

### UNIT OUTLINE

#### I. DEFINING TERMS

- A. HOW OSCILLATIONS 'ARE' WAVES

#### II. EQUATIONS OF SHM

- A. DISPLACEMENT AND VELOCITY  
B. ACCELERATION - THE DEFINING EQUATION OF SHM

#### III. ENERGY IN SHM

#### IV. DAMPING

- A. ENERGY LOSS IN SHM  
B. DEGREES AND TYPES OF DAMPING

#### V. RESONANCE

- A. FORCED OSCILLATIONS AND RESONANCE  
B. 'GOOD' AND 'BAD' RESONANCE

### FROM THE IB DATA BOOKLET

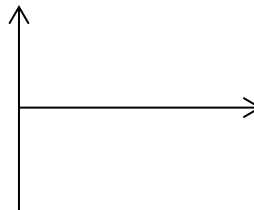
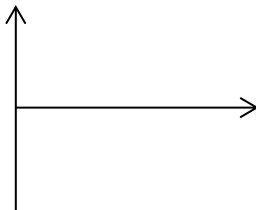
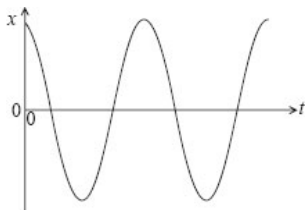
$$\omega = \frac{2\pi}{T} \quad \begin{array}{ll} x = x_0 \sin \omega t; & x = x_0 \cos \omega t \end{array} \quad \begin{array}{l} v = \pm \omega \sqrt{(x_0^2 - x^2)} \\ E_{K(\max)} = \frac{1}{2} m \omega^2 x_0^2 \end{array}$$
$$\begin{array}{ll} v = v_0 \cos \omega t; & v = -v_0 \sin \omega t \end{array} \quad \begin{array}{l} E_K = \frac{1}{2} m \omega^2 (x_0^2 - x^2) \\ E_T = \frac{1}{2} m \omega^2 x_0^2 \end{array}$$

### WHAT YOU SHOULD BE ABLE TO DO AT THE END OF THIS TOPIC

- Describe examples of oscillations and SHM through the defining equation of SHM
- Define the terms amplitude, displacement, angular frequency, frequency, period, and phase in the context of SHM.
- Apply and use the equations of SHM in different circumstances
- Discuss properties of SHM from graphs
- Recognize that in SHM there is a continuous transformation of energy, between KE and EPE
- Describe the effect of aperiodic external force and damping on an oscillating system
- Understand the meaning of resonance and give examples of its occurrence

**HOMEWORK PROBLEMS:**

1. The graph below the variation with time  $t$  of the displacement  $x$  of a particle undergoing simple harmonic motion. On the two blank graphs sketch the variation with time of the velocity  $v$  and the acceleration  $a$  of the particle.



2. The graph to the right shows the variation with displacement  $x$  of the acceleration  $a$  of a body.

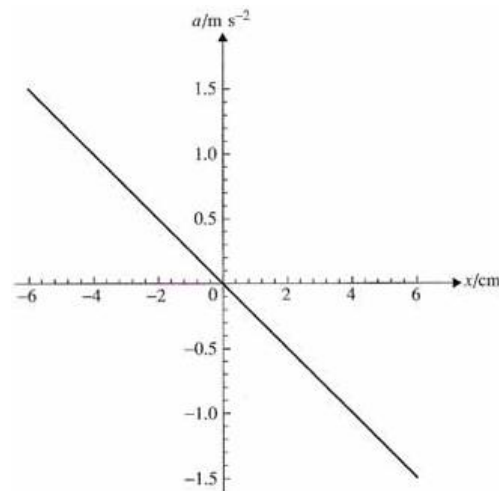
a) Explain how it may be deduced that the body executes SHM.

b) Use the graph to determine the period of the oscillations.

**[1.3 s]**

c) Determine the maximum speed of the body during the oscillations.

**[0.30 ms<sup>-1</sup>]**



3. The displacement of a particle executing SHM is given by  $y = 5.0\cos(2t)$  where  $y$  is in millimeters and  $t$  is in seconds. Calculate:

a) the initial displacement of the particle.

**[5.0 mm]**

b) the displacement at  $t = 1.2$  s.

**[-3.7 mm]**

c) the time at which the displacement first becomes  $-2.0$  mm.

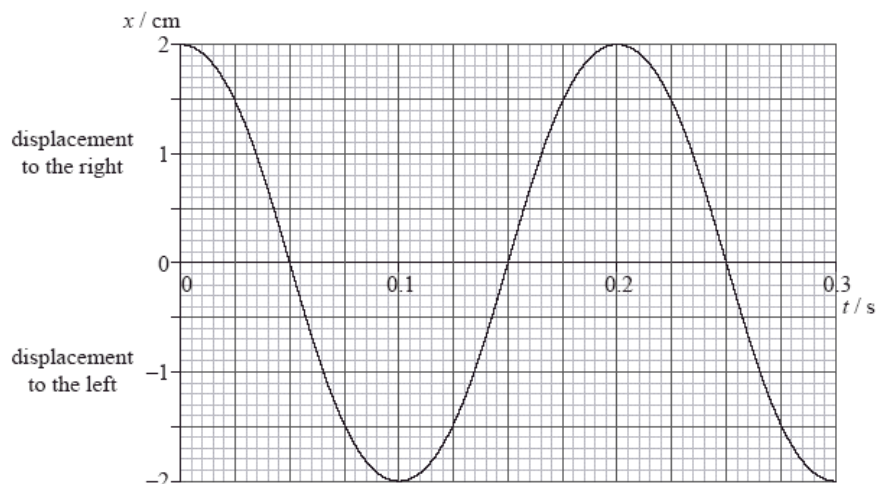
**[0.99 s]**

d) the displacement when the velocity of the particle is  $6.0 \text{ mm s}^{-1}$ .

**[±4.0 mm]**

4. A longitudinal wave travels through a medium from left to right. Graph 1 shows the variation with time  $t$  of the displacement  $x$  of a particle P in the medium.

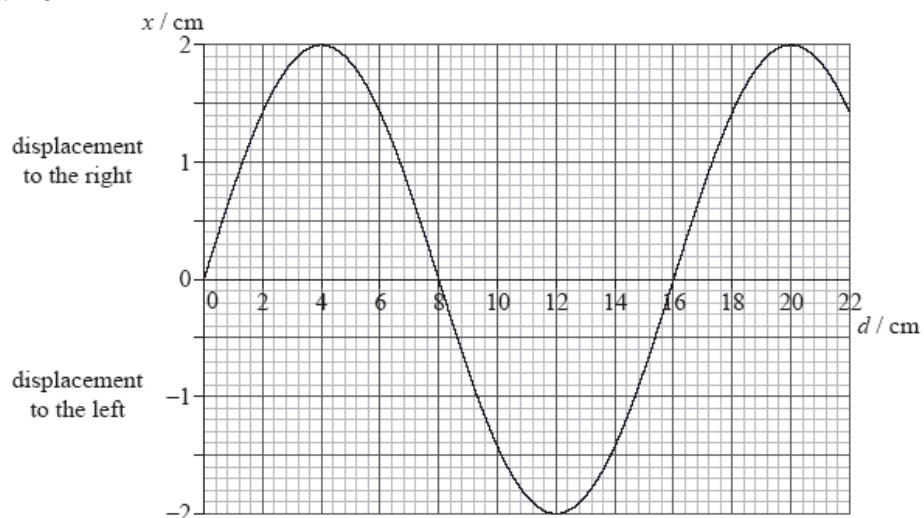
**Graph 1**



- a) calculate the magnitude of the particle's maximum acceleration. [20 m s<sup>-2</sup>]
- b) calculate the particle's speed at  $t = 0.12$  s. [0.37 m s<sup>-1</sup>]
- c) state the particle's direction of motion at  $t = 0.12$  s. [to the right]

Graph 2 shows the variation with position  $d$  of the displacement  $x$  of particles in the medium at a particular instant of time.

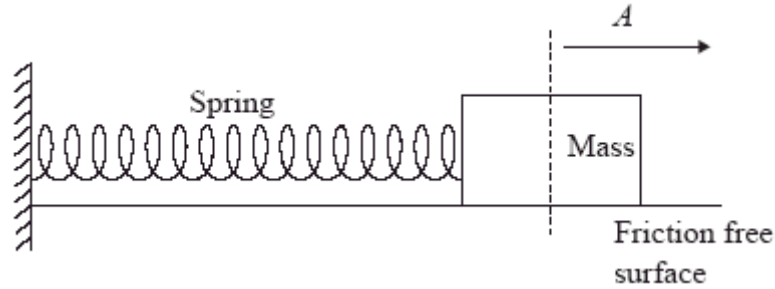
**Graph 2**



Determine for the longitudinal wave, using graph 1 and graph 2,

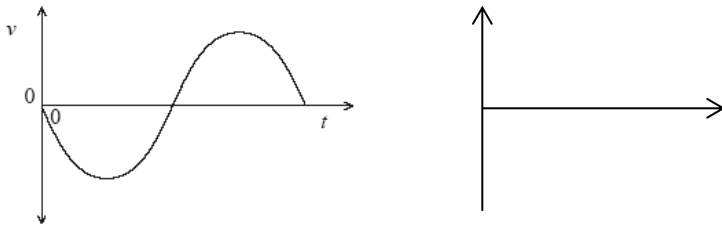
- d) the frequency. [5.0 Hz]
- e) the speed. [0.80 m s<sup>-1</sup>]

5. A mass on the end of a horizontal spring is displaced from its equilibrium position by a distance  $A$  and released. Its subsequent oscillations have total energy  $E$  and time period  $T$ .

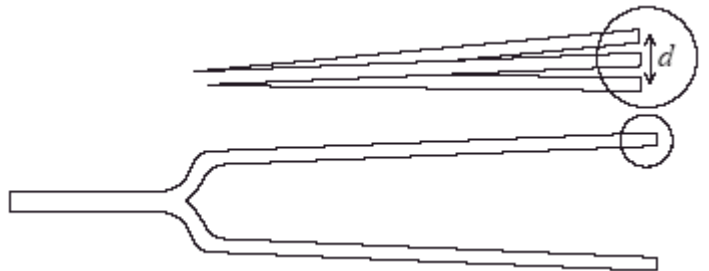


An identical mass is attached to an identical spring. The maximum displacement is  $2A$ . Assuming this spring obeys Hooke's law, determine the new time period and the total energy of the oscillations in terms of these given variables. **[T, 4E]**

6. The graph shows how the velocity  $v$  of an object undergoing simple harmonic motion varies with time  $t$  for one complete period of oscillation. On the blank axis provided, sketch how the total energy  $E$  varies with time  $t$ .



7. A tuning fork is sounded and it is assumed that each tip vibrates with simple harmonic motion. The extreme positions of the oscillating tip of one fork are separated by a distance  $d$ .



- a) State, in terms of  $d$ , the amplitude of vibration. **[d/2]**

- b) On the axes below, sketch a graph to show how the displacement of one tip of the tuning fork varies with time. On your graph, label the time period  $T$  and the amplitude  $A$ .



c) The frequency of oscillation of the tips is 440 Hz and the amplitude of oscillation of each tip is 1.2 mm. Determine the maximum

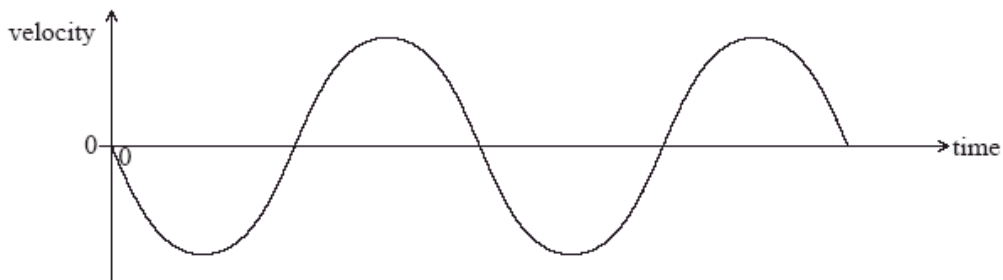
i) linear speed of a tip.

[3.3 ms<sup>-1</sup>]

ii) acceleration of a tip.

[9.2 x 10<sup>3</sup> m s<sup>-2</sup>]

d) The sketch graph below shows how the velocity of a tip varies with time.



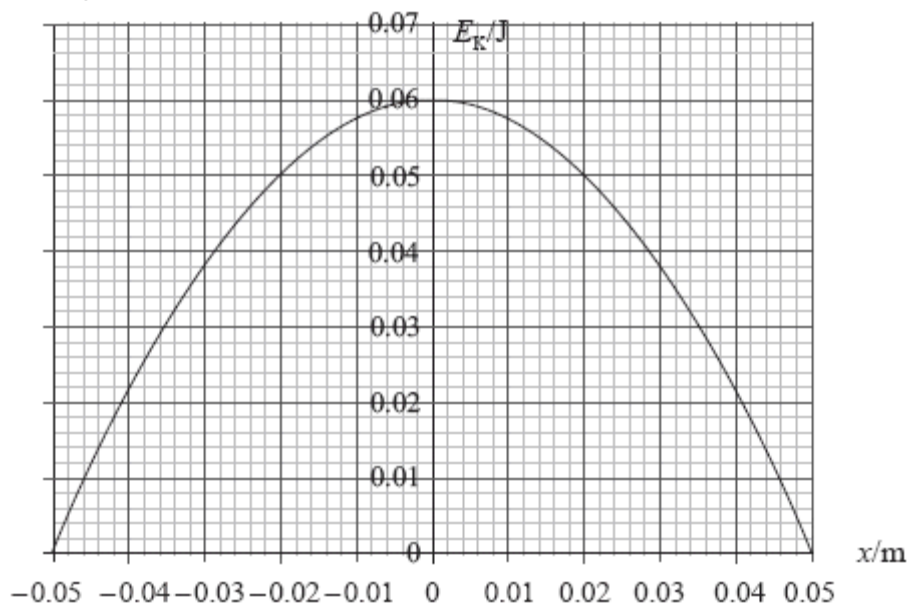
On the axes, sketch a graph to show how the acceleration of the tip varies with time.

e) In practice, the motion of the tips of the tuning fork is damped. Suggest **one** reason why the motion of the tips is damped.

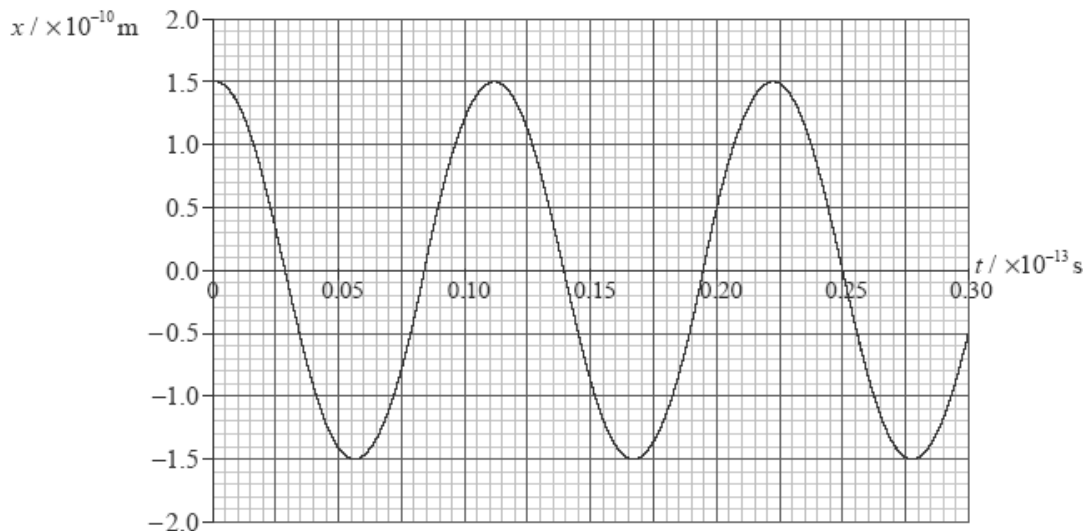
8. A particle of mass  $m$  that is attached to a light spring is executing simple harmonic motion in a **horizontal direction**.

a) State the condition relating to the **net force** acting on the particle that is necessary for it to execute simple harmonic motion.

- b) The graph shows how the kinetic energy  $E_k$  of the particle in (a) varies with the displacement  $x$  of the particle from equilibrium.



- c) Using the axes above, sketch a graph to show how the **potential energy** of the particle varies with the displacement  $x$ .
- d) The mass of the particle is 0.30 kg. Use data from the graph to show that the frequency  $f$  of oscillation of the particle is 2.0 Hz.
- e) The particles of a medium  $M_1$  through which a transverse wave is travelling oscillate with the same frequency and amplitude as that of the particle in (b).
- Describe, with reference to the propagation of energy through the medium, what is meant by a transverse wave.
  - The speed of the wave is  $0.80 \text{ m s}^{-1}$ . Calculate the wavelength of the wave. **[0.40 m]**
9. In a simple model of a methane molecule, a hydrogen atom and the carbon atom can be regarded as two masses attached by a spring. A hydrogen atom is much less massive than the carbon atom such that any displacement of the carbon atom may be ignored. The graph below shows the variation with time  $t$  of the displacement  $x$  from its equilibrium position of a hydrogen atom in a molecule of methane.



a) The mass of hydrogen atom is  $1.7 \times 10^{-27} \text{ kg}$ . Use data from the graph above

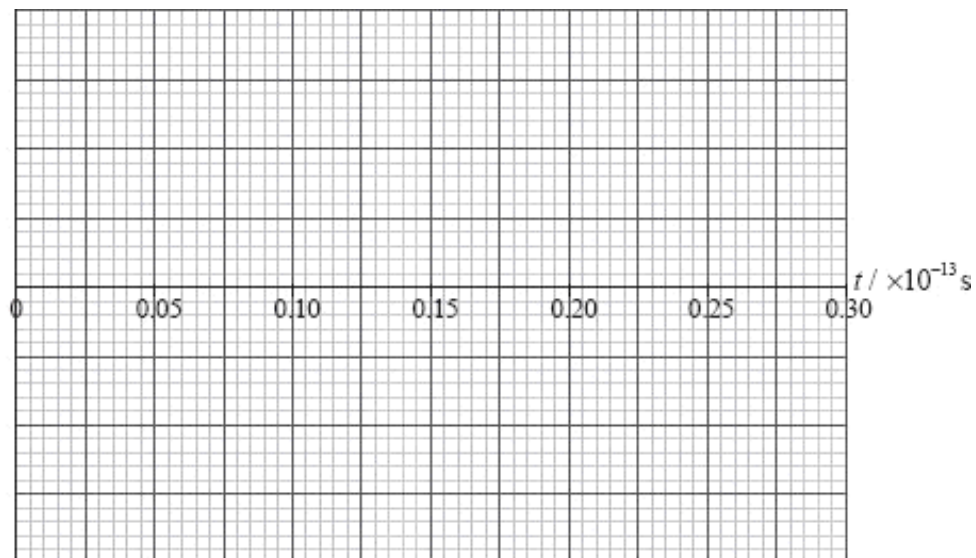
i) to determine its amplitude of oscillation.

[ $1.5 \times 10^{-10} \text{ m}$ ]

ii) to show that the frequency of its oscillation is  $9.1 \times 10^{13} \text{ Hz}$ .

iii) to show that the maximum kinetic energy of the hydrogen atom is  $6.2 \times 10^{-18} \text{ J}$ .

b) On the grid, sketch a graph to show the variation with time  $t$  of the velocity  $v$  of the hydrogen atom for one period of oscillation starting at  $t = 0$ . (There is no need to add values to the velocity axis.)



- c) Assuming that the motion of the hydrogen atom is simple harmonic, its frequency of oscillation  $f$  is given by the expression

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m_p}},$$

where  $k$  is the force per unit displacement between a hydrogen atom and the carbon atom and  $m_p$  is the mass of a proton.

- i) Show that the value of  $k$  is approximately  $560 \text{ N m}^{-1}$ .
- ii) Estimate, using your answer to (i), the maximum acceleration of the hydrogen atom.  
**[5.0 x 10<sup>19</sup> m s<sup>-2</sup>]**

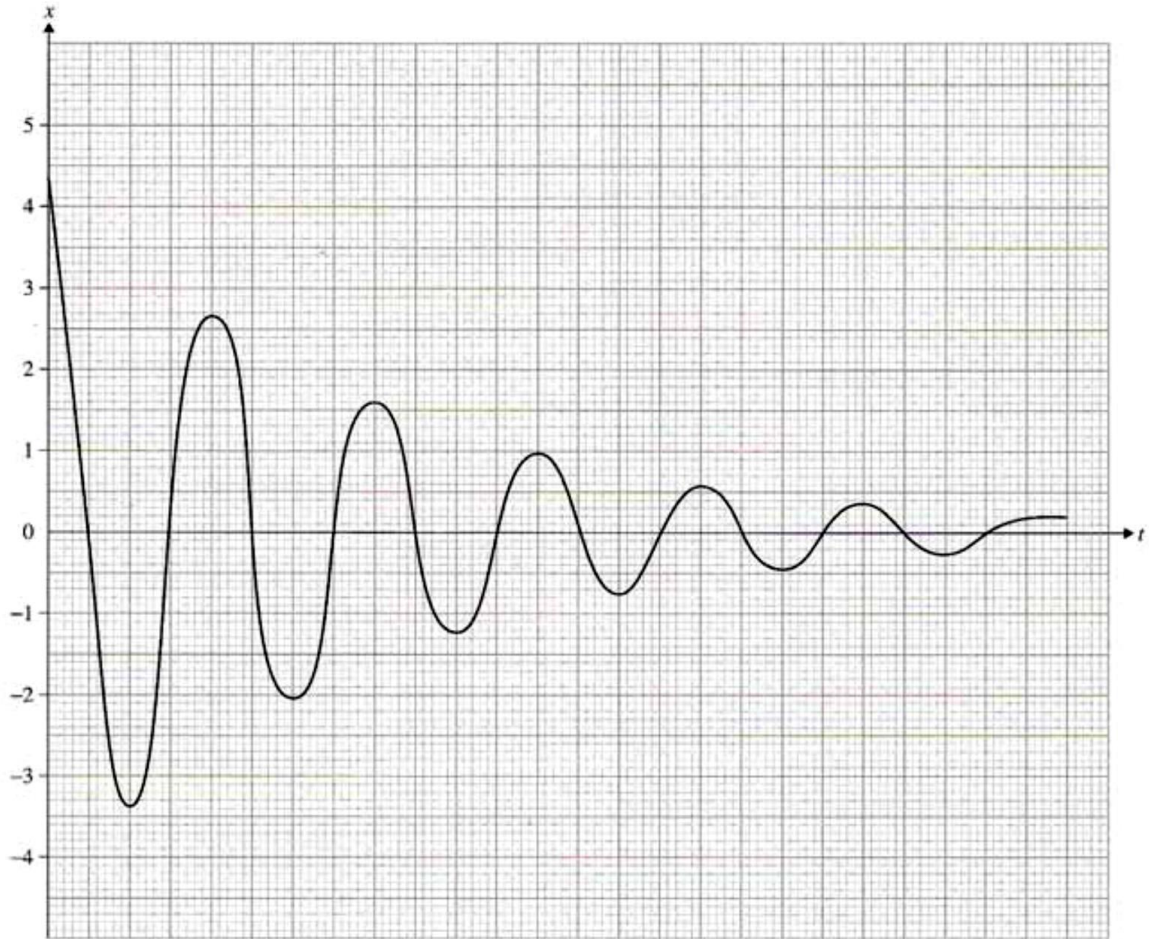
- d) Methane is classified as a **greenhouse gas**.

- i) Discuss what is meant by a greenhouse gas (you may have to do some research on this one).

- ii) Electromagnetic radiation of frequency  $9.1 \times 10^{13} \text{ Hz}$  is in the infrared region of the electromagnetic spectrum. Suggest, based on the information given in (a)(ii), why methane is classified as a greenhouse gas .



10. The graph shows the variation with time  $t$  of the displacement  $x$  of a particle undergoing SHM that is under-damped.



- a) By making measurements on the diagram, determine whether the ratio of successive amplitudes stays constant.
  
- b) The amount of energy stored in the oscillation is proportional to the square of the amplitude. Determine, for these oscillations, the amount of energy lost in one oscillation as a percentage of the energy stored in the previous oscillation.
  
- c) On the same axes, draw a graph to show the changes, if any, to the variation of displacement if the amount of damping were to increase (but still keep the oscillations under-damped).