NAME: ______________________________ **DATE:** __________ PHYSICS - Mr. Smith

Richard Feynman and the "Principle of Least Action'

(from Richard Feynman, a Life in Science by John and Mary Gribbin)

One of the most important principles in physics is the conservation of energy – the total amount of energy associated with the ball (in this example) stays the same. Some of this energy is in the form of gravitational potential energy, which depends on its height above the surface of the Earth (strictly speaking, on its distance from the centre of the Earth). When the ball rises, it gains gravitational potential energy; when it falls, it loses some of this energy. The only other relevant form of energy possessed by the ball is its energy of motion, or kinetic energy. Higher speeds correspond to greater kinetic energy. At the moment the ball leaves the thrower's hand, it has a lot of kinetic energy because it is moving fast. As it rises, some of this kinetic energy is lost, traded for gravitational potential energy, and it slows down. At the top of its trajectory, it has minimum kinetic energy and maximum potential energy, then as it falls down the other side of the curve it gains kinetic energy and loses potential energy. But the *total*, the sum of (kinetic + potential) energy is always the same.

All this Feynman knew. But what he didn't know was that given the time taken for the journey, the trajectory followed by the ball is always the one for which the *difference*, kinetic energy *minus* potential energy, added up all along the trajectory, is the *least*. This is the Principle of Least Action, a property involving the whole path.

Looking at the curved line on a blackboard representing the flight of the ball, you might think, for example, that you could make it take the same time for the journey by throwing it slightly more slowly, in a flatter arc, more nearly a straight line; or by throwing it faster along a longer trajectory, looping higher above the ground. But nature doesn't work that way. There is only one possible path between two points for a given amount of time taken for the flight. Nature 'chooses' the path with the least action – and this applies not just to the flight of a ball, but to any kind of trajectory, at any scale. Mr Bader didn't work out the numbers involved, or ask Feynman to work them out. He just told him about the principle, a deep truth which impressed the high school student in his final year before going on to college.

It's worth a slight detour to give another example of the principle at work, this time in the guise of the Principle of Least Time, because it is so important both to science and to Feynman's career. This version of the story involves light. It happens that light travels slightly faster through air than it does through glass.* Either in air or glass, light travels in straight lines – an example of the Principle of Least Time, because, since a straight line is the shortest distance between two points, that is the quickest way to get from A to B. But what if the journey from A to B starts out in air, and ends up inside a glass block? If the light still travelled in a single straight line, it would spend a relatively small amount of time moving swiftly through air, then a relatively long time moving slowly through glass. It turns out (see Figure 1) that there is a unique path which enables the light to take the least time on its journey, which involves travelling in a certain straight line up to the edge of the glass, then turning and travelling in a different straight line to its destination. The light seems to 'know' where it is going, apply the Principle of Least Action, and 'choose' the optimum path for its journey.

The connection between mathematics and physics highlighted by the Principle of Least Action reinforced a growing fascination that Richard had had with this area of science right through high school. While working with radio receivers, building his own circuits and working out how to tune them, he had come across equations describing the behaviour of these practical objects that involved the Greek pi, the ratio of the circumference of a circle to its diameter. Although there were circular (or cylindrical) coils in these circuits, it is also possible to work with square coils, and pi came into the equations whatever the shape of the coils. There was some deep link between physics and mathematics, which Feynman did not understand, but which intrigued him. Although still

*The famous 'ultimate speed limit' from relativity theory is the speed of light *in a vacuum*, which is greater still.

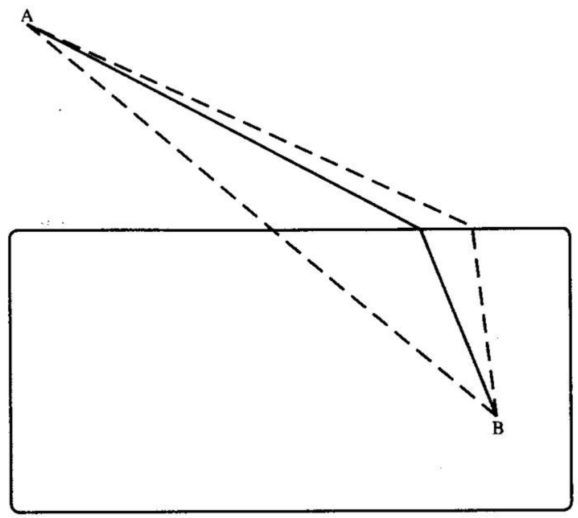

Figure 1. Light travels faster through air than through glass. So the quickest journey from A to B that is partly through air and partly through glass is not the (dotted) straight line from A to B, but there is a unique 'path of least time' made up of two straight lines. This is a special case of the Principle of Least Action at work. The dotted lines to the right show an example of a path that takes longer than the path of least time (solid lines).

known as a whiz at maths, his fascination was really with physics.

We have emphasized the role of science in young Richard's life because it was, indeed, the main thing in his life. He went through the educational system in what seemed, superficially, a conventional way, but actually learned his science for himself, outside the system (including teaching himself about relativity theory from books while still in high school). He found school boring, but sailed through examinations with ease, appearing, in that respect, to have been a model student.