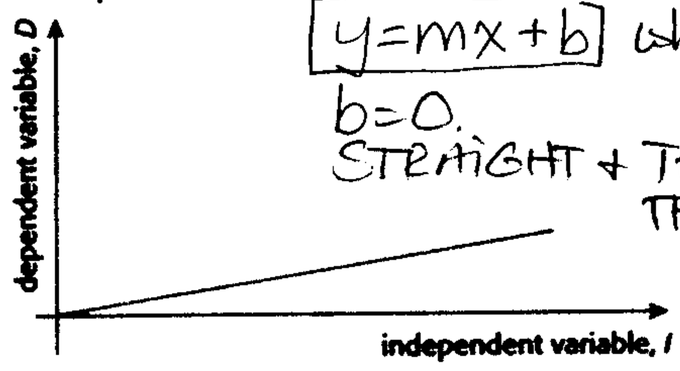
1.2 GRAPHING IN PHYSICS — *HW SOLUTIONS*

1. For each graph A-H, suggest a relationship between y and x; in other words, guess the function y(x).

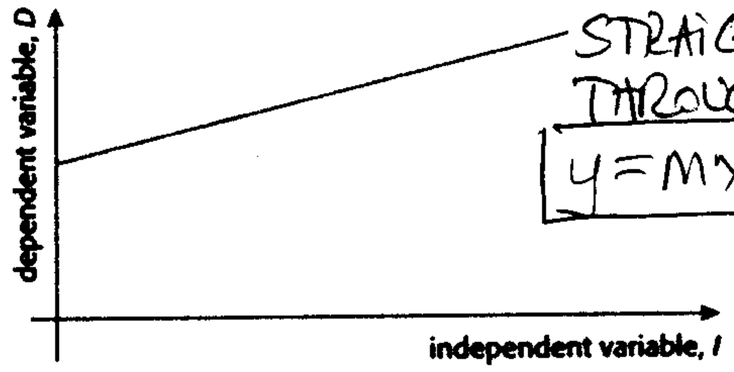
★ = common in IB physics

Graph A:



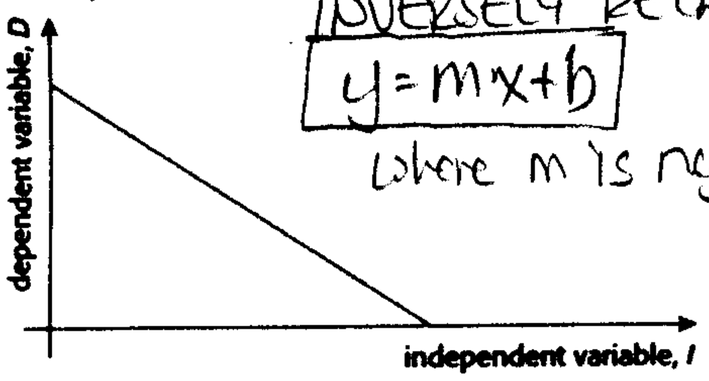
PROPORTIONAL:
$y = mx + b$ where
$b = 0$.
STRAIGHT + THROUGH THE ORIGIN

Graph B:



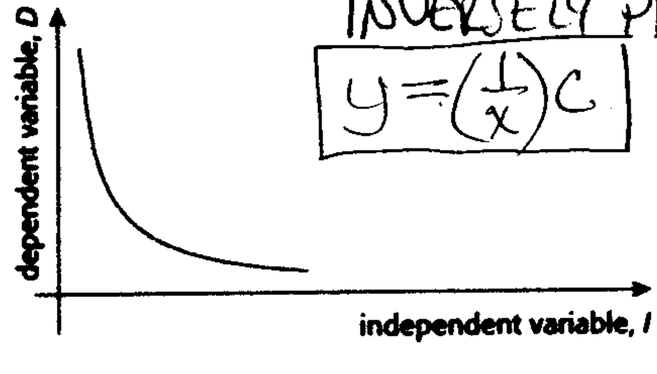
NOT PROPORTIONAL
STRAIGHT BUT NOT THROUGH ORIGIN
$y = mx + b$

Graph C:



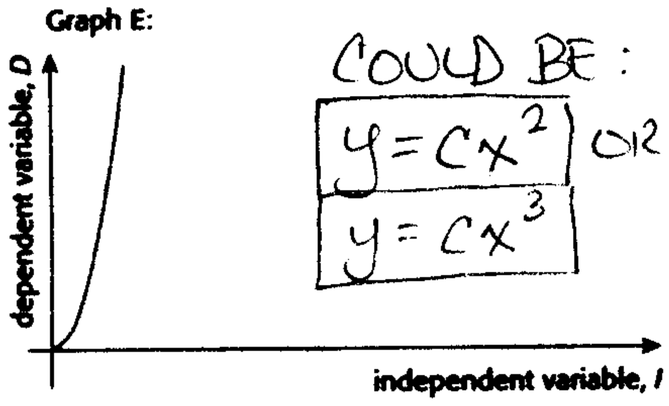
INVERSELY RELATED
$y = mx + b$
where m is negative

Graph D:



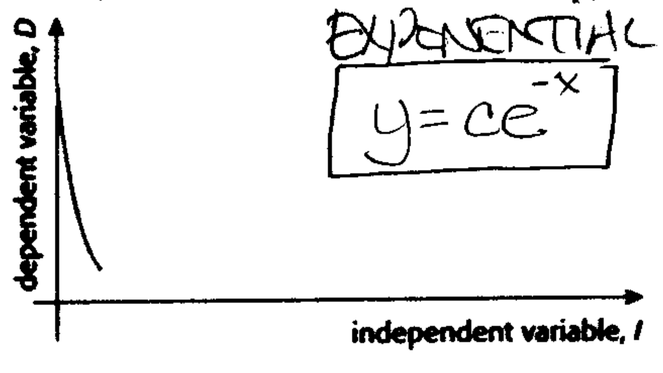
INVERSELY PROPORTIONAL
$y = \left(\frac{1}{x}\right)C$

Graph E:



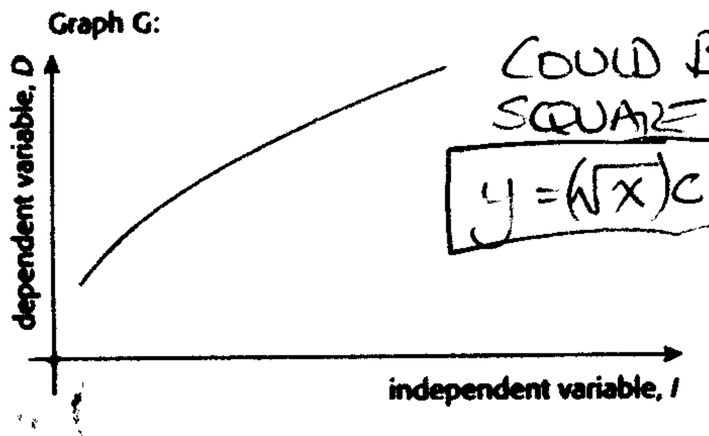
COULD BE:
$y = Cx^2$ OR
$y = Cx^3$

Graph F:



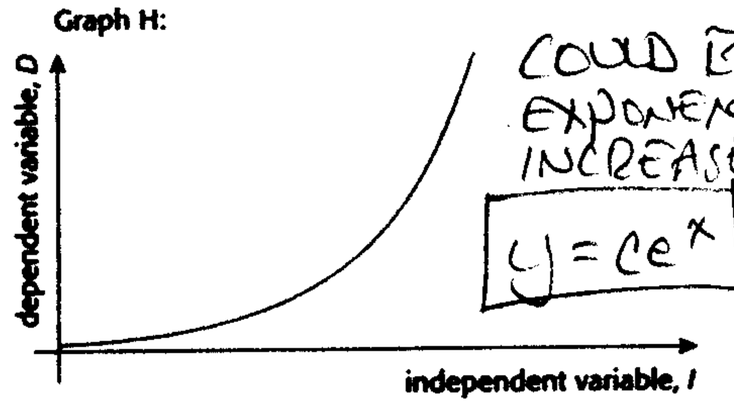
COULD BE EXPONENTIAL DECREASE
$y = Ce^{-x}$

Graph G:



COULD BE SQUARE ROOT:
$y = (\sqrt{x})C$

Graph H:



COULD BE EXPONENTIAL INCREASE
$y = Ce^{x}$

2. A clam farmer has been keeping records concerning the water temperature and the number of clams developing from fertilized eggs. The data is recorded to the right. Graph the data and determine the optimum temperature for clam development.

SEE ATTACHED SHEET

H_20 T / °C	Number of clams
15	75
20	90
25	120
30	140
35	75
40	40
45	15
50	0

3. According to Charles' Law, the volume of a gas decreases as the temperature of the gas decreases. A sample of gas was collected at 100 °C and then cooled. The changes in the volume of the sample were measured, and the data is shown below.

T / °C	V / ml
100	317
80	297
60	288
40	278
30	252
20	243
10	236
0	233
-10	227
-30	202

a) Graph the data, allowing space for extrapolation.
(X RANGE : -300 to 120 ; Y RANGE : -50 to 350)

b) The temperature at which the volume of the gas reaches zero is the theoretical temperature of absolute zero. From your graph, determine the T of absolute zero in °C.

SEE ATTACHED SHEET.

-265°C

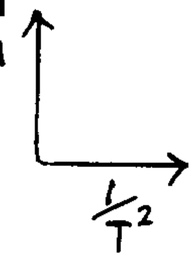
4. A particle is moving in a circular path of radius *r*. The time taken for one complete revolution is *T*. The acceleration *a* of the particle is given by the expression $a = \frac{4\pi^2 r}{T^2}$. Assuming *a* and *T* are experimental variables (*r* is held constant), what would you graph in order to get a straight line and what would the slope of the line represent?

$a = \frac{4\pi^2 r}{T^2} \Rightarrow$ rewrite as: $y = mx + b$

$a = (4\pi^2 r)\frac{1}{T^2} + 0$

SO GRAPH a VS. $\frac{1}{T^2}$.

The slope gives the quantity $4\pi^2 r$ from which r could be found.



5. The speed of sound *v* in a gas is related to the pressure *P* of the gas by the expression $v = \sqrt{kP}$ where *k* is a constant. Which variables should be plotted in order to produce a straight line graph with the slope equal to *k*? What are the units of this slope?

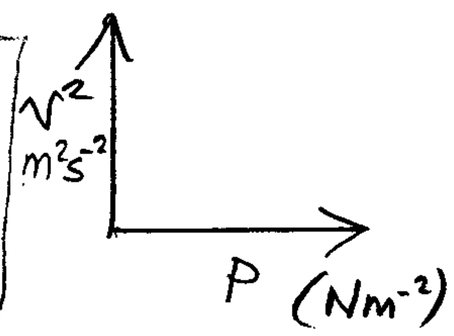
$v = \sqrt{kP} \Rightarrow$ Rewrite as: $y = mx + b$

$v = C\sqrt{P} + 0$

OR $v^2 = kP$

So, GRAPH v^2 VS. P

The slope gives the constant k in $m^4N^{-1}s^{-2}$



6. The variation with speed *v* of the force *F* acting on an object is given by the expression $F = pv^2 + qv$ where *p* and *q* are constants. What quantity should be plotted on the *y*-axis of a graph and what should be plotted on the *x*-axis in order to give a straight-line graph, and what are the units of the resulting gradient?

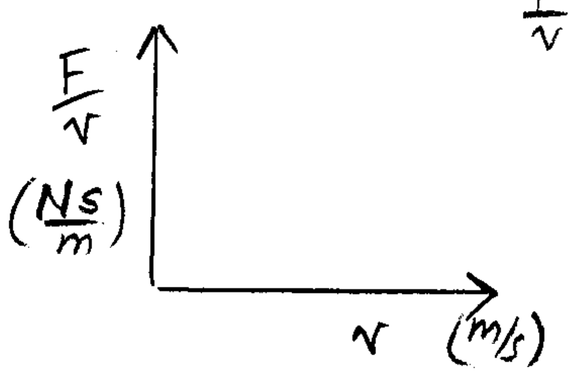
$F = pv^2 + qv$

$= v(pv + q) \Rightarrow \frac{F}{v} = pv + q$

OF THE FORM: $y = mx + b$

$\frac{F}{v} \cdot \frac{1}{v} = \left(\frac{Ns}{m} \cdot \frac{s}{m}\right)$

$= \frac{Ns^2}{m^2}$



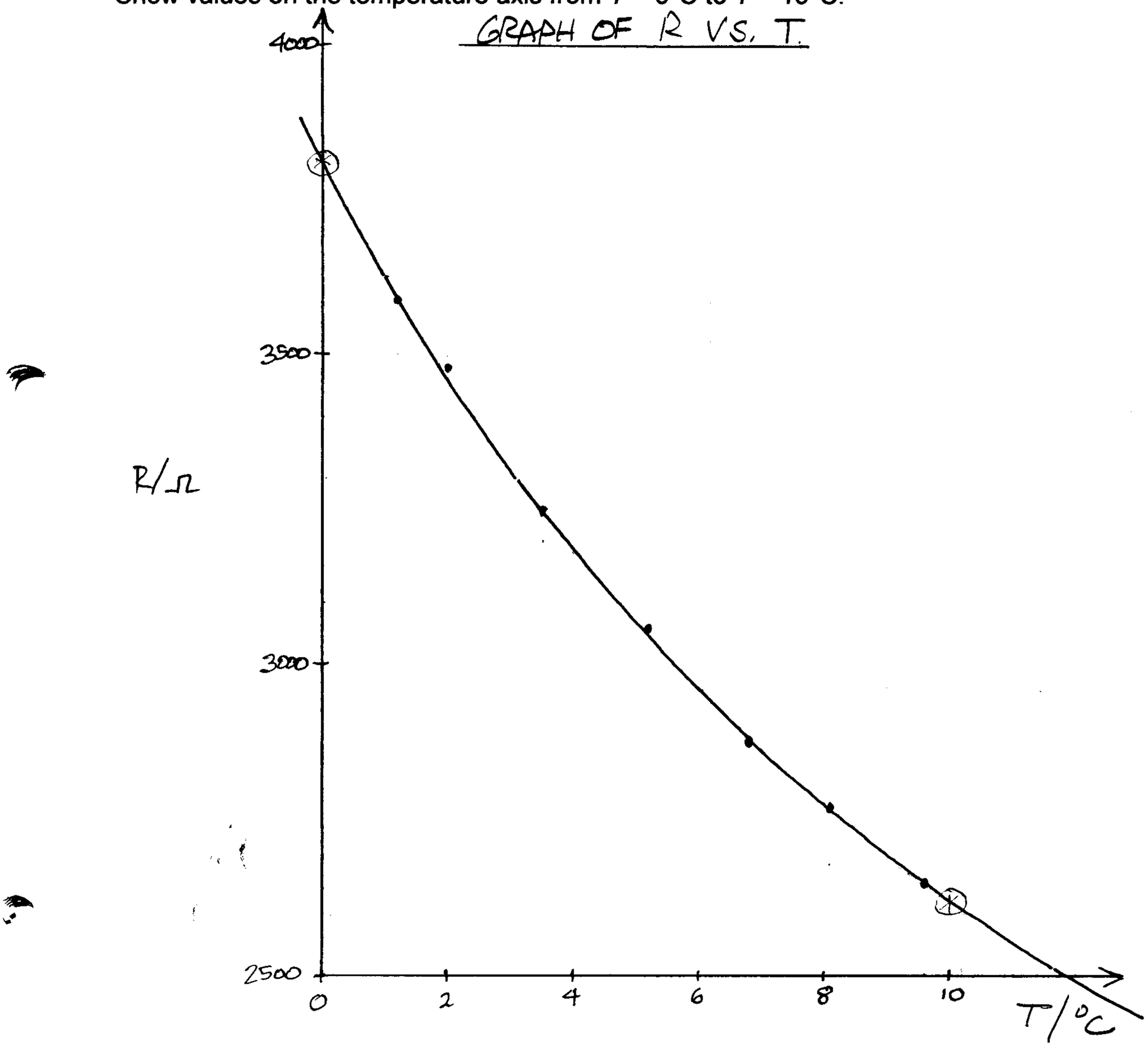
where $y = \frac{F}{v}$, $m = p$, $x = v$, and $b = q$.

∴ GRAPH $\frac{F}{v}$ VS. v. THE SLOPE WOULD GIVE YOU A VALUE FOR THE CONSTANT p, THE UNITS OF WHICH ARE Ns^2m^{-2}

7. The table below gives values of the resistance R of an electrical component for different values of its temperature *T*.

***T*/°C**	1.2	2.0	3.5	5.2	6.8	8.1	9.6
***R*/Ω**	3590	3480	3250	3060	2880	2770	2650

a) On the grid below, plot a graph to show the variation with temperature *T* of the resistance *R*. Show values on the temperature axis from *T* = 0°C to *T* = 10°C.



b) Draw a curve that best fits the points you have plotted. Extend your curve to cover the temperature range from 0°C to 10°C.

c) Use your graph to determine the resistance at 0°C and at 10°C.

AT 0°C : 3800Ω

AT 10°C : 2620Ω

8. The thickness of the annual rings of a tree indicate what type of environmental situation was occurring at the time of its development. A thin ring, usually indicates a rough period of development, such as lack of water, forest fires, or a major insect infestation. A thick ring indicates just the opposite. Below are data for trees in two different forests:

Age of the tree / years	FOREST A: Average thickness of the annual rings / cm	FOREST B: Average thickness of the annual rings / cm
10	2.0	2.2
20	2.2	2.5
30	3.5	3.6
35	3.0	3.8
50	4.5	4.0
60	4.3	4.5

a) Graph this data, and draw in the best fit curve to the data points. SEE ATTACHED SHEET

b) What was the average thickness of the annual rings of 40 year old trees in Forest A?

c) Based on this data, what can you conclude about Forest A and Forest B?

9. The data table contains time and position data for a bicyclist riding from home on a trip. SEE ATTACHED SHEET

a) Graph the data and connect the points with straight lines.

b) From your graph, estimate the *position* of the bicycle at 45 min. and 75 min.

c) From your graph, estimate the *speed* of the bicycle at 40 min. and 65 min.

d) During what time interval is the bicycle going the fastest? Is the bicycle ever at rest? If so, during what time interval(s)?

Time / min	Distance from home / km
0	0
10	1.0
20	2.0
30	4.5
40	4.5
50	4.5
60	7.0
70	8.5
80	8.5
90	6.5

10. The data table shows experimental time and position data for a moving object (distance measured from a stationary point). SEE ATTACHED SHEET

a) Make a proper line graph of this data on the grid below. Carefully draw the best fit line through your data.

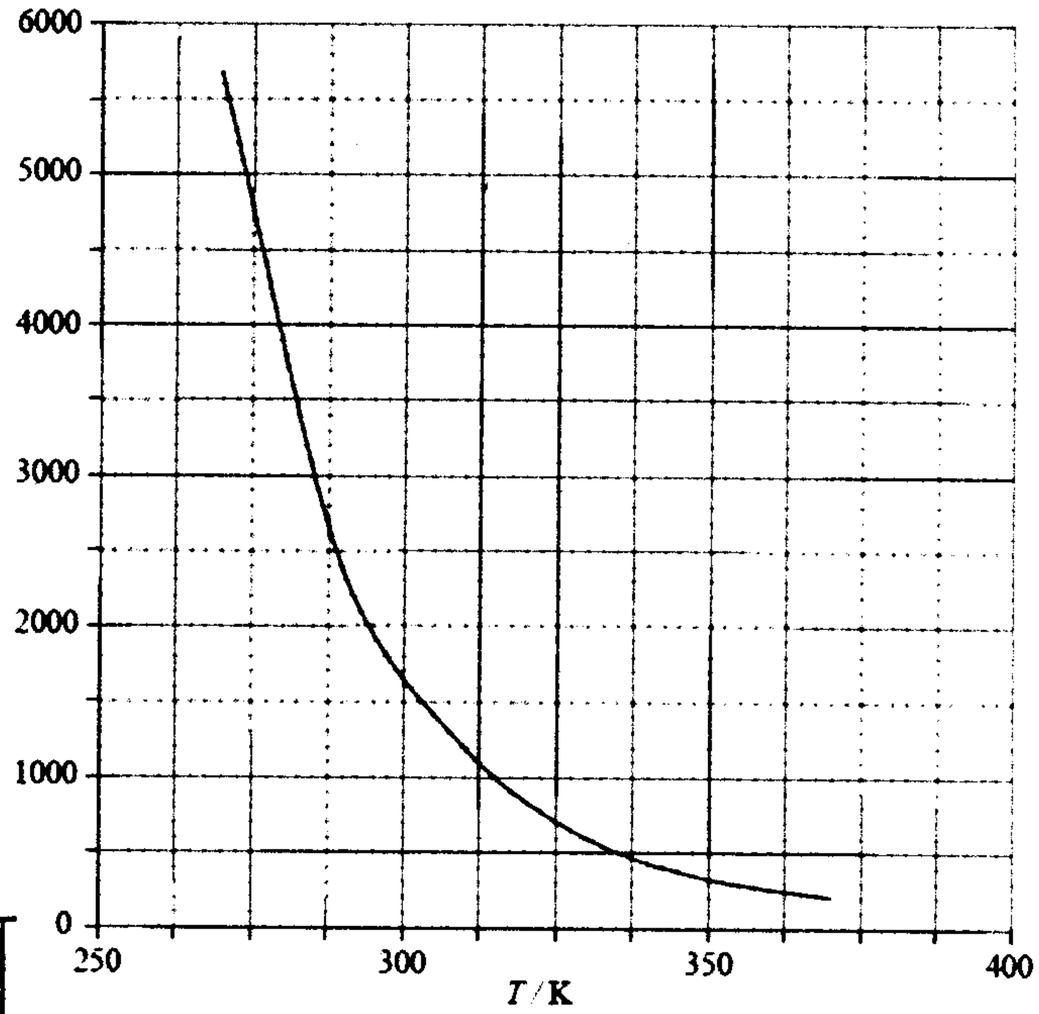
b) From your graph, estimate the *position* of the object at 3.5 s and 10.5 s.

c) From your graph, estimate the *speed* of the object at 4.5 s and 6.0 s.

d) Briefly explain the movement of this object in a short paragraph. What do you think the object might be?

Time / s	Position / cm
0.0	8.0
1.0	4.1
2.0	-4.0
3.0	-7.9
4.0	-4.0
5.0	4.2
6.0	8.1
7.0	4.0
8.0	-4.1
9.0	-8.0
10.0	-3.9
11.0	4.1
12.0	7.9

11. The graph shows the resistance R vs. temperature T for an electrical component. A student hypothesizes that resistance is inversely proportional to temperature. Use the graph to determine whether the student is correct.



IF INVERSELY PROPORTIONAL, THEN $R \propto \frac{1}{T}$ OR $R = \left(\frac{1}{T}\right)C$

AND THUS RT = Constant.

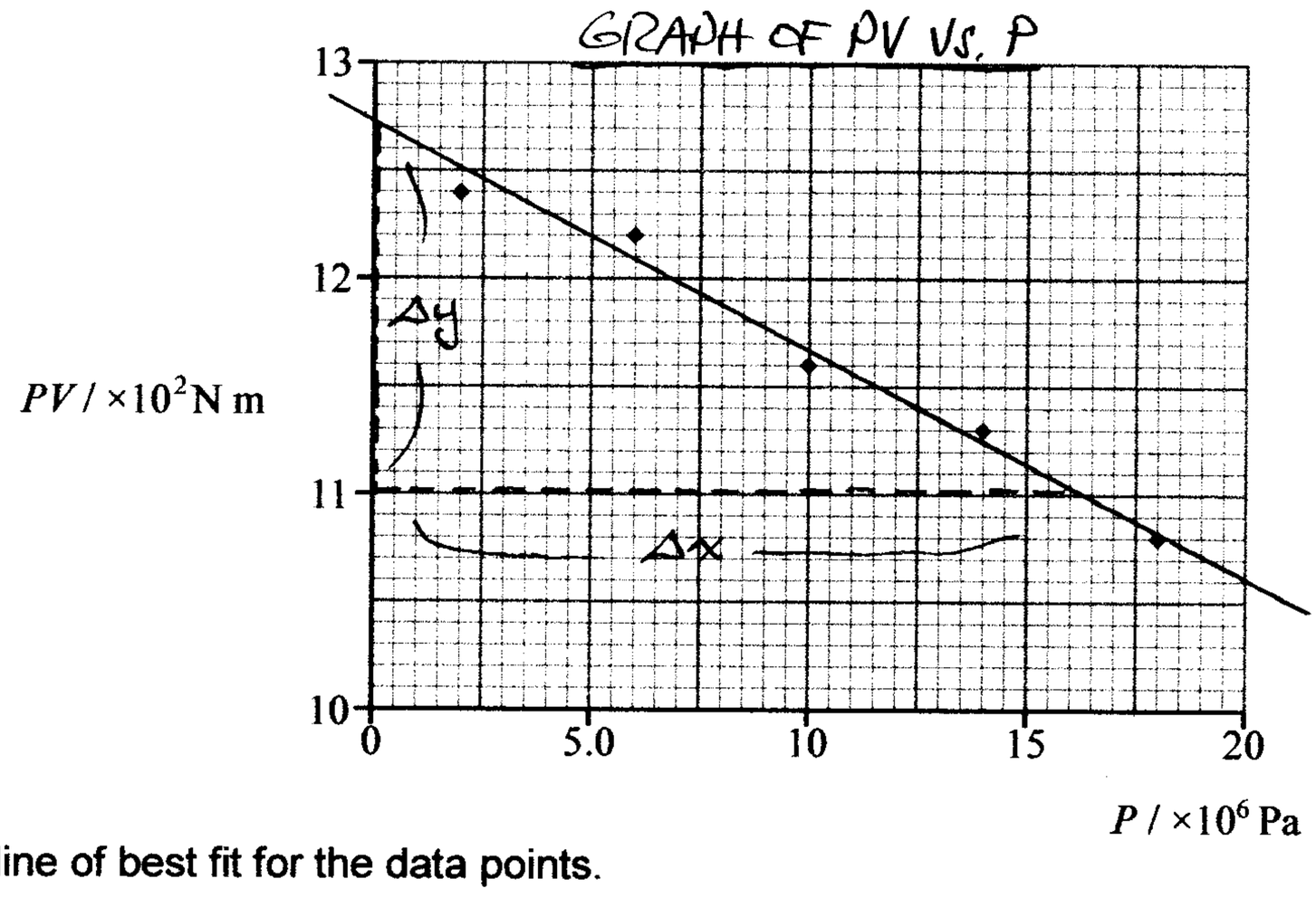
T	R	RT
275	4750	1.3×10^6
300	1650	5.0×10^5
325	700	2.3×10^5
350	350	1.2×10^5

RT IS NOT CONSTANT, SO STUDENT IS NOT CORRECT.

12. At high pressures, a real gas does not behave as an ideal gas. For a certain range of pressures, it is suggested that the relation between the pressure P and volume V of one mole of the gas at constant temperature is given by the equation

$$PV = A + BP \qquad \text{where } A \text{ and } B \text{ are constants.}$$

In an experiment to measure the deviation of nitrogen gas from ideal gas behaviour, 1 mole of nitrogen gas was compressed at a constant temperature of 150 K. The volume V of the gas was measured for different values of the pressure P. A graph of the product PV of pressure and volume was plotted against the pressure P and is shown below.



a) Draw a line of best fit for the data points.

b) Use your best fit line to determine the values (with units) of the constants A and B in the equation

$$PV = A + BP$$

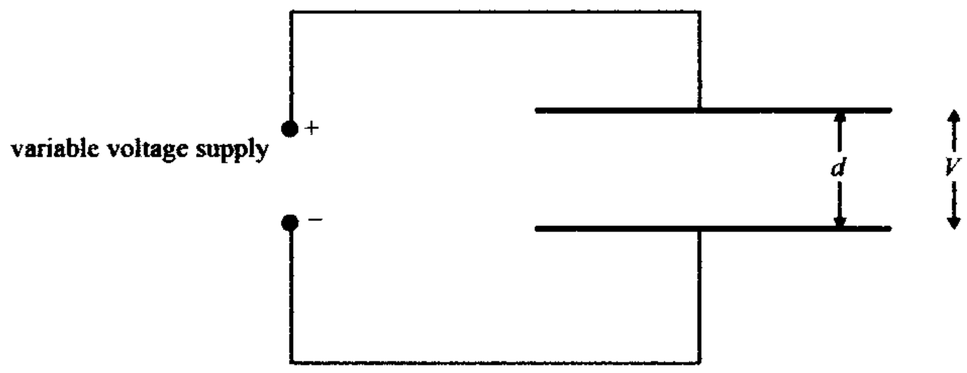
$$y = b + mx$$

So, A = y-intercept $= 12.7\times10^2\ \text{N m} = \boxed{1.3\times10^3\ \text{N m} = A}$

B = slope $= \frac{\Delta y}{\Delta x} = \frac{(11-12.7)\times10^2}{(15.6-0)\times10^6} = \frac{-1.7\times10^2}{15.6\times10^6} = \boxed{1.1\times10^{-5}\ \text{N m Pa}^{-1} = B}$

5

13. The diagram shows two parallel conducting plates connected to a variable voltage supply. The plates are of equal areas and are a distance *d* apart.



The charge *Q* on one of the plates is measured for different values of the potential difference *V* applied between the plates. The values obtained are shown in the table below. Uncertainties in the data are not included.

Assume that the graph goes through the origin. Later you will learn that if V=0, Q=0.

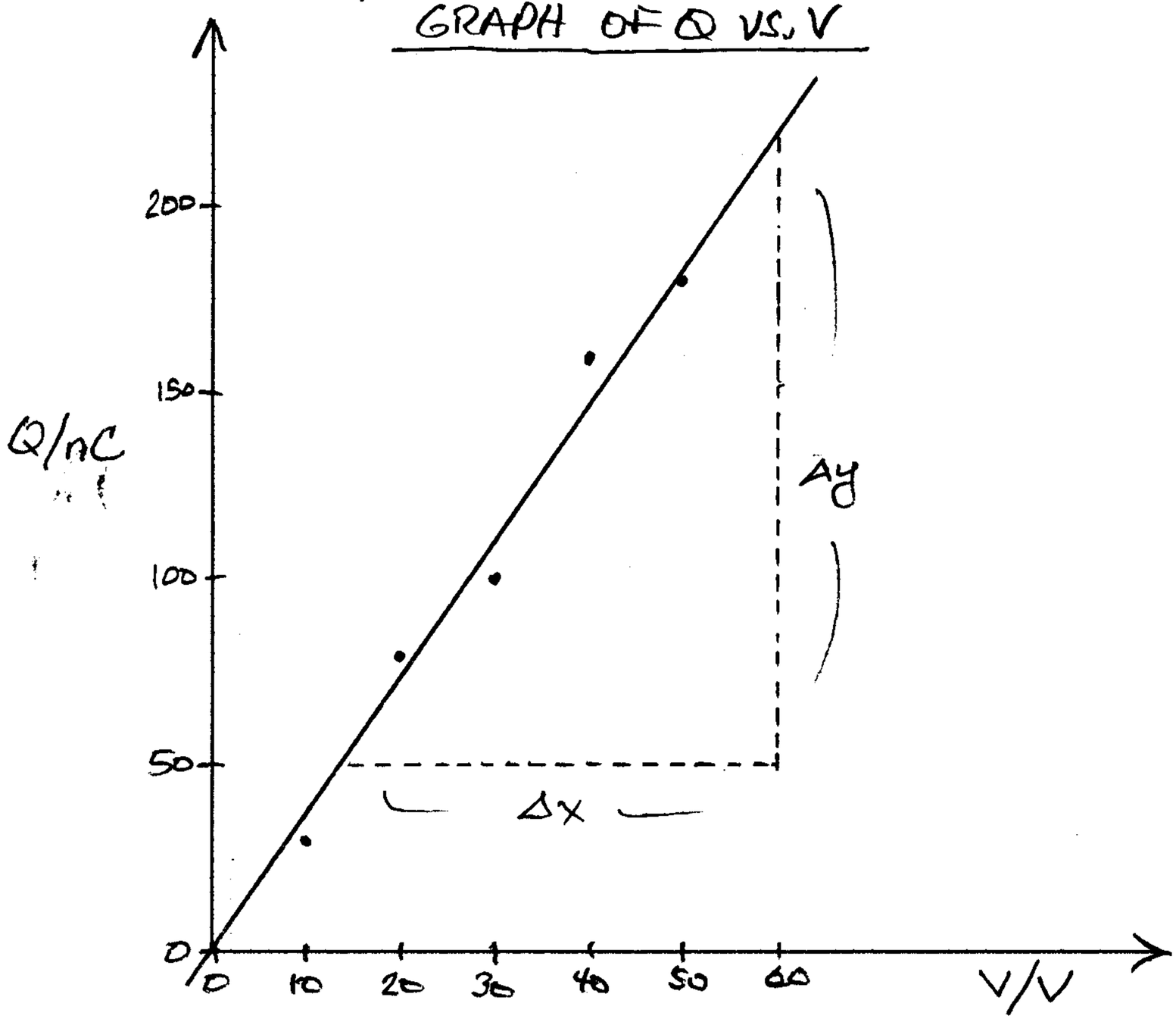
V / V	Q / nC
10.0	30
20.0	80
30.0	100
40.0	160
50.0	180

a) Plot a graph of *V* (*x*-axis) against *Q* (*y*-axis) and draw the line of best fit for the data points.

b) Determine the gradient of your best-fit line.

c) The gradient of the graph is a property of the two plates and is known as *capacitance*. Deduce the units of capacitance.

GRAPH OF Q VS. V

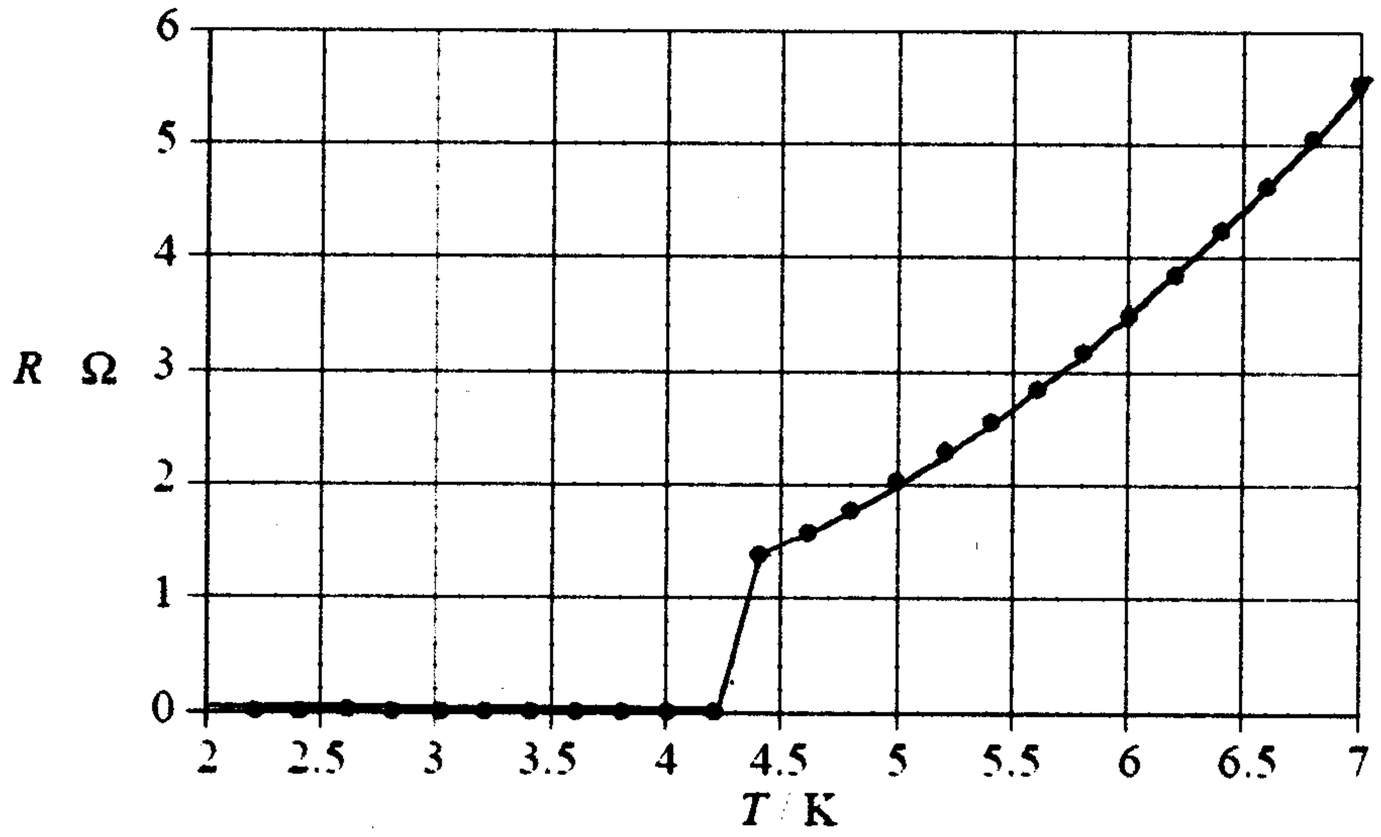


* NOTE:
The units as shown here are $\frac{\Delta y}{\Delta x} = \frac{nC}{V}$.
But express as $\frac{C}{V}$.

$$\text{SLOPE} = \frac{\Delta y}{\Delta x} = \frac{(220-50)\times 10^{9}}{(60-14)}\ \frac{C}{V} = \frac{170\times 10^{-9}}{46}\ \frac{C}{V}$$

$$= 3.7\times 10^{-9}\ C V^{-1}$$

14. The resistance R of a sample of mercury was measured as a function of temperature T of the sample. The sample was cooled, and data points were taken at temperature intervals of 0.2 K and graphed:



a) The hypothesis is that R is proportional to T for $T > 4.5$ K. Suggest whether the data supports this hypothesis and why.

NO; The graph in that region is not a straight line.

b) Draw a line of best fit through the data.

c) State the value of R for which the rate of change of R of the sample with T is least.

At R=.0Ω, the line has no slope, so no change.

d) At a temperature T_c, the resistance suddenly becomes zero. State a range for the possible values of T_c based on the graph.

4.2-4.4 K

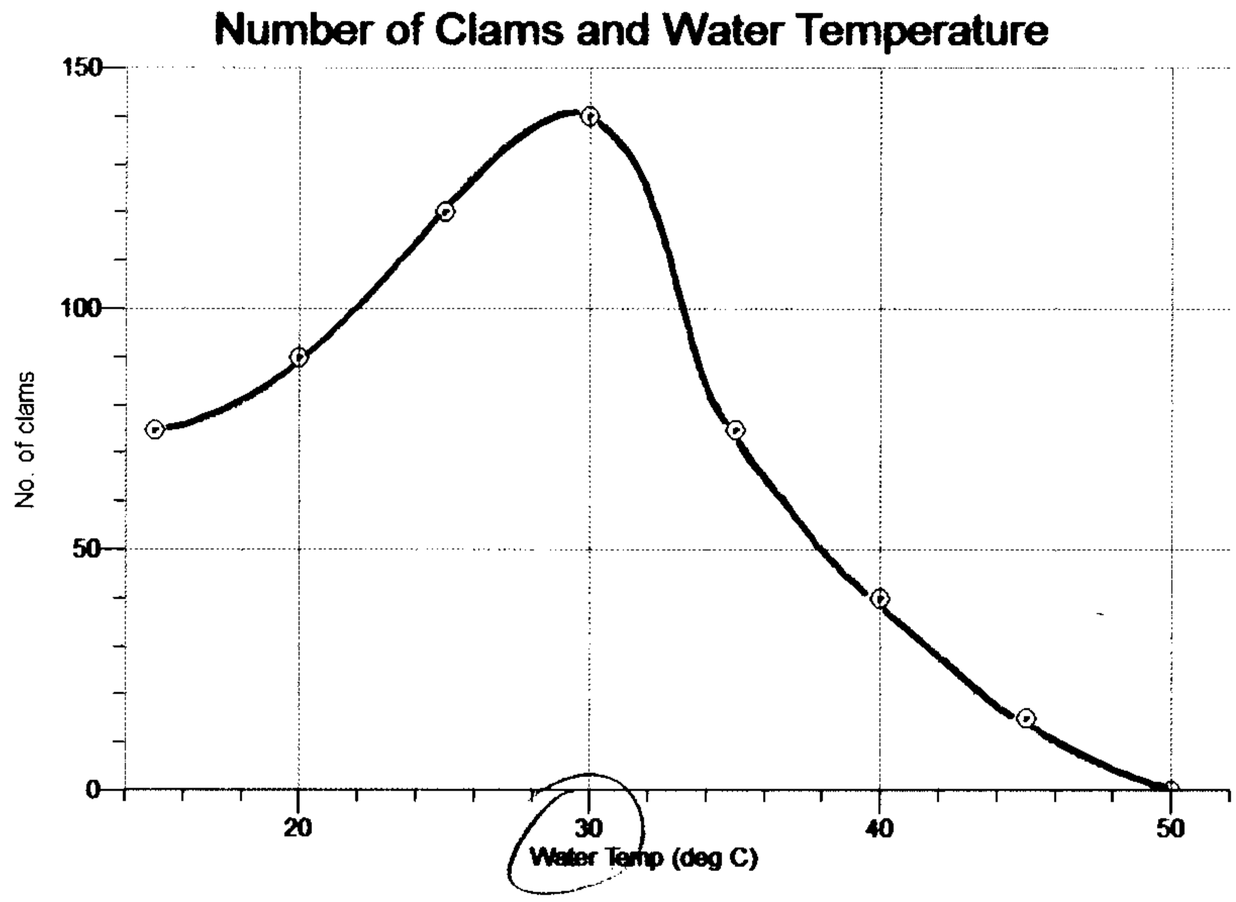
e) Outline 2 reasons why you would not use the data in the graph to determine an accurate value for R at room temperature.

① The data are for low temps, well below room temp.

② There is no reason to assume the trends continue to room temp.

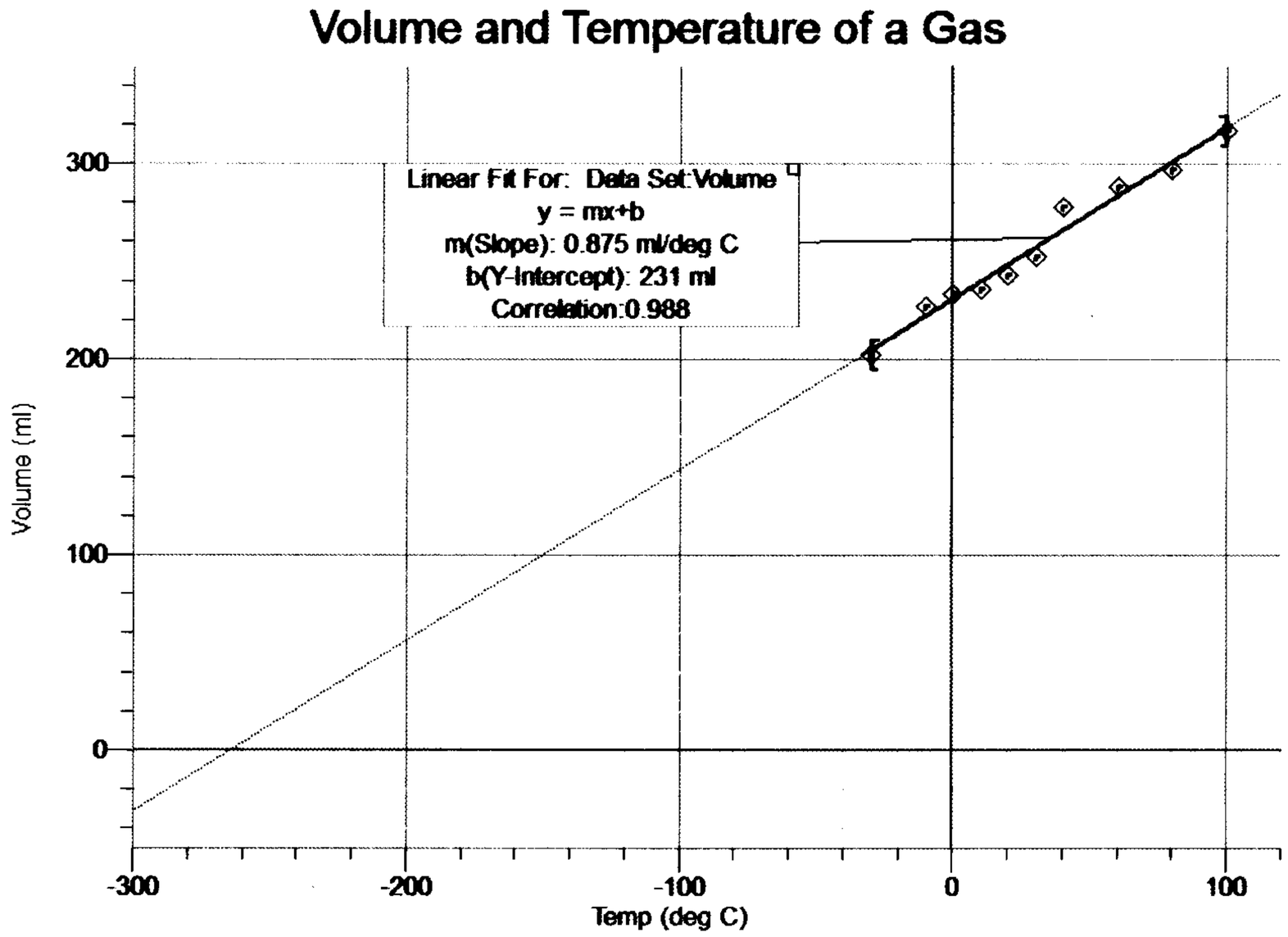
... others?

2.



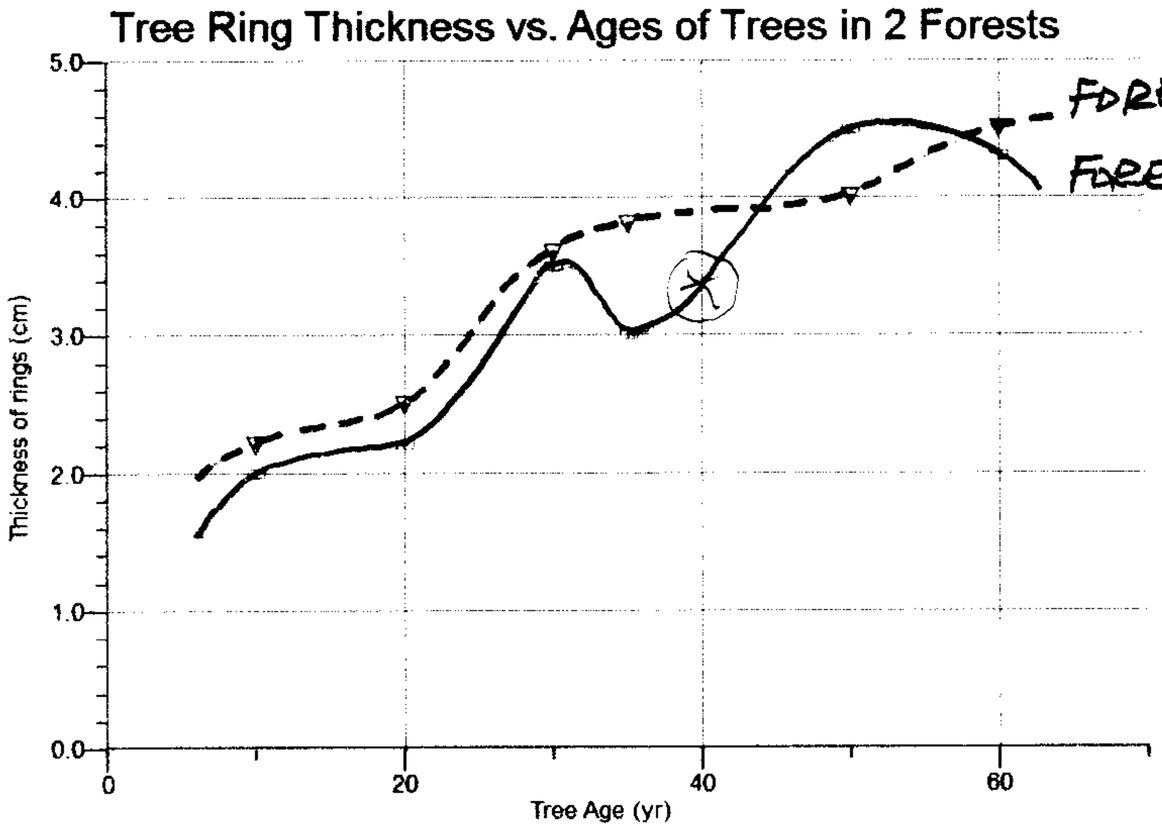
The optimum temperature for clam development is around 30°C, as shown by the graph.

3.



The line crosses the 0 ml horizontal at **T = -265°C**. Note that this is very close to the given value of absolute zero, 0 K = -273°C.

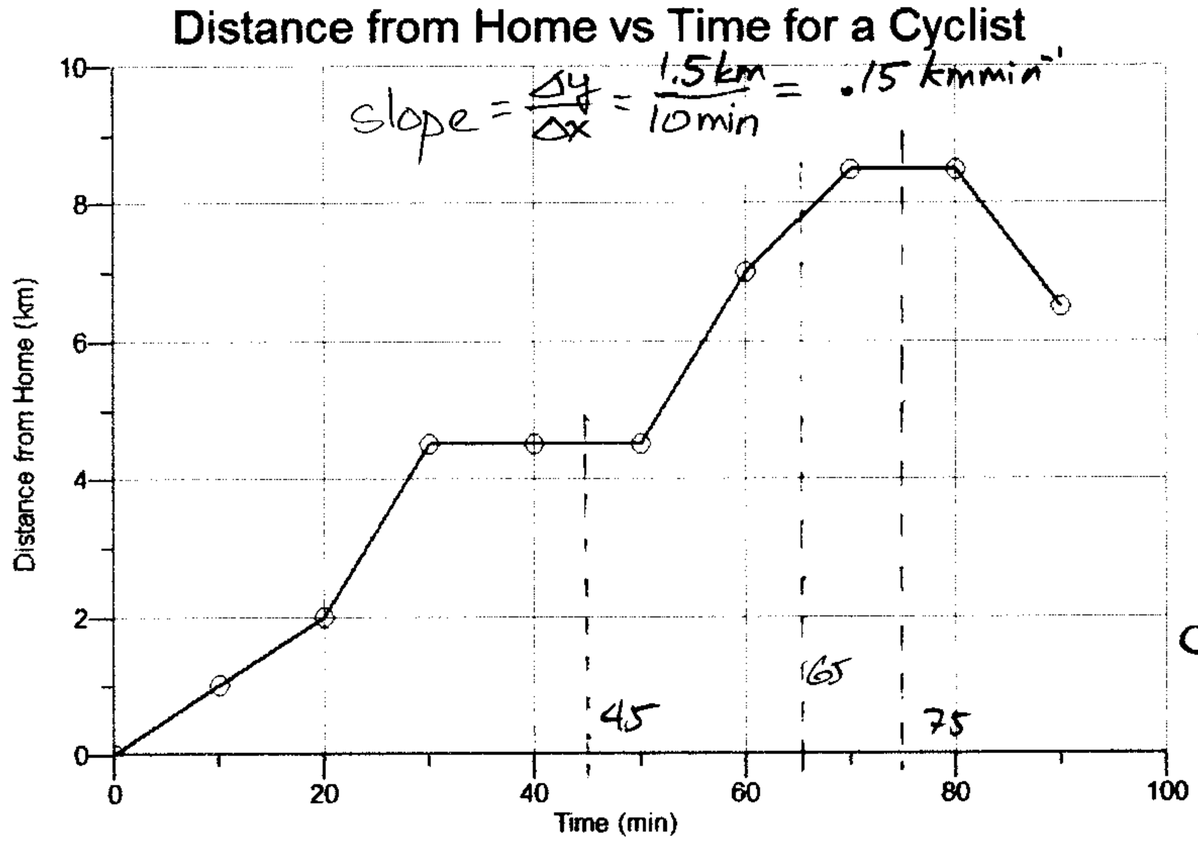
8.



b) 40 year-old trees in forest A are about **3.2 cm**.

c) Forest A has a more variable climate. At 30 years, for example, it must have been wet; between 30 and 40 years dry, etc...

9.



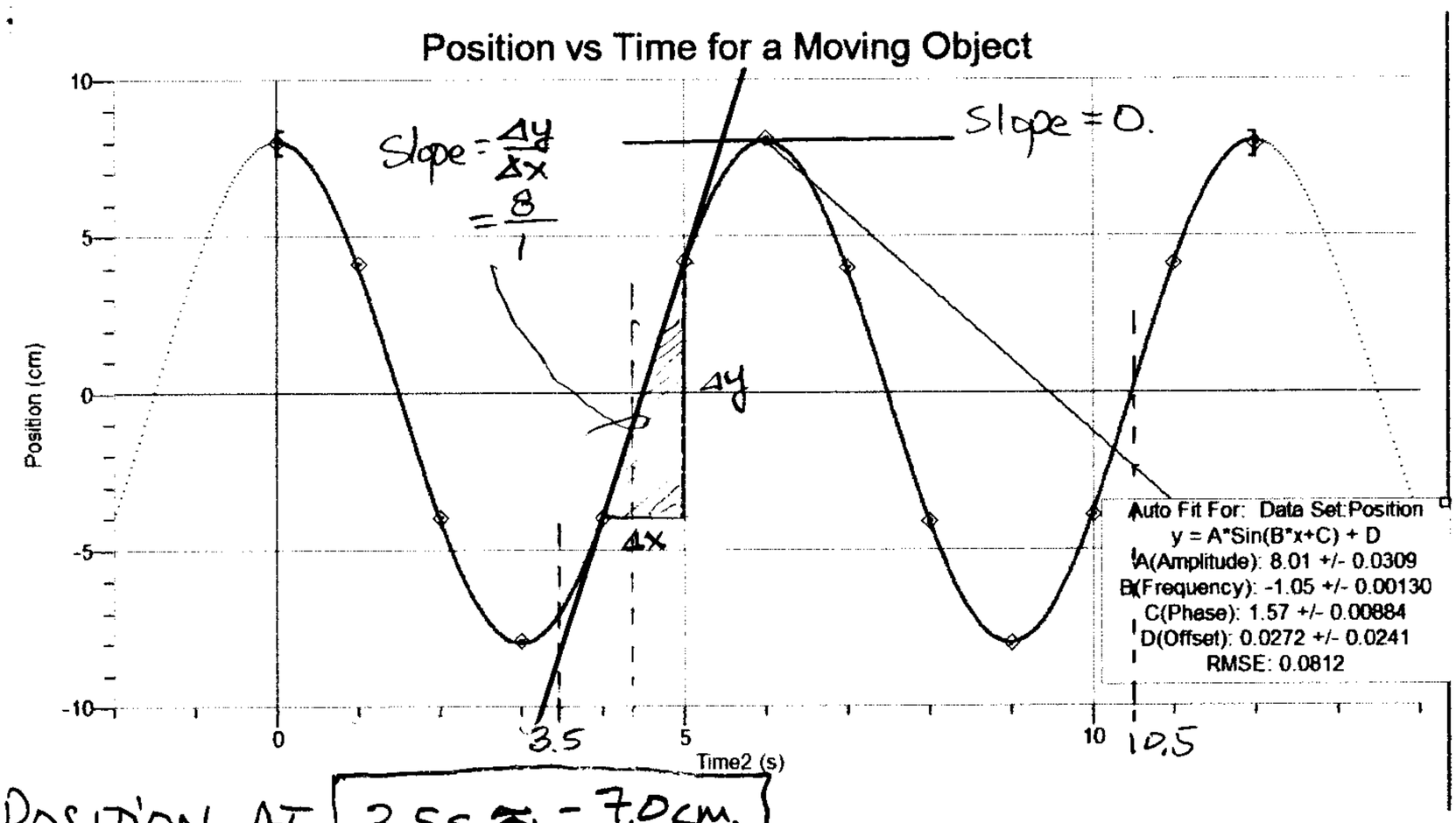
b) At 45min: **4.5km**
At 75min: **8.5km**

c) Speed is the slope of the s-t graph.
At 40min: **0 kmmin⁻¹**
At 65min: **0.15 kmmin⁻¹**

d) FASTEST: From 20-30min
From 50-60min
From 80-90min.

AT REST: 30-50min,
70-80 min

10.



b) POSITION AT **3.5s ≈ -7.0cm.**
10.5s ≈ 0.0cm

c) SPEED = SLOPE AT **4.5s ≈ 8.0 cms⁻¹**
6.0s ≈ 0.0 cms⁻¹

d) MOVEMENT IS SINUSOIDAL. IT IS THE MOTION OF A PENDULUM.