### 1.2 UNCERTAINTIES AND ERRORS

| SL/HL |  |
| :--- | :--- |
| $\frac{\text { Required: }}{\text { READ Hamper pp 4-10 }}$ | $\frac{\text { Supplemental: }}{\text { READ Tsokos, pp 8-12 }}$ |

## REMEMBER TO....

$\checkmark \quad$ Work through all of the 'example problems' in the texts as you are reading them
$\checkmark \quad$ Refer to the IB Physics Guide for details on what you need to know about this topic
$\checkmark \quad$ Refer to the Study Guides for suggested exercises to do each night
$\checkmark \quad$ First try to do these problems using only what is provided to you from the IB Data Booklet
$\checkmark \quad$ Refer to the solutions/key ONLY after you have attempted the problems to the best of your ability

## UNIT OUTLINE

I. UNCERTAINTY AND ERROR IN MEASUREMENT
A. PRECISION AND ACCURACY
B. RANDOM AND SYSTEMATIC ERRORS
C. REPORTING A SINGLE MEASUREMENT
D. REPORTING YOUR BEST ESTIMATE OF A MEASUREMENT

## II. UNCERTAINTY IN CALCULATED RESULTS

A. ABSOLUTE AND PERCENTAGE UNCERTAINTIES
B. DETERMINING FINAL UNCERTAINTIES IN STATED RESULTS
C. WHAT TO DO WITH MAX AND MIN LINES

## FROM THE IB DATA BOOKLET

If: $y=a \pm b$
If: $y=\frac{a b}{c}$
then: $\Delta y=\Delta a+\Delta b$

$$
\text { then: } \frac{\Delta y}{y}=\frac{\Delta a}{a}+\frac{\Delta b}{b}+\frac{\Delta c}{c}
$$

If: $y=a^{n}$
then: $\frac{\Delta y}{y}=\left|n \frac{\Delta a}{a}\right|$

## WHAT YOU SHOULD BE ABLE TO DO AT THE END OF THIS TOPIC

- State and identify the various types of errors that may arise when measuring quantities
- Recognize the difference between accuracy and precision
- State uncertainties as absolute uncertainties and percentage uncertainties
- Determine the uncertainties in calculated results
- Depict uncertainties in graphs as error bars
$\square$ Determine uncertainties in the gradient (using max and min lines) and intercepts of a linear graph


## HOMEWORK PROBLEMS:

1. What are the width and its absolute uncertainty of the object being measured in the sketch below?

2. The analogue voltmeter shown measures a voltage on a scale of zero to 5 volts. What is the measured voltage and what is the absolute uncertainty shown here?

3. Four students measure the same length of string and their results are as follows:
$L_{1}=38.6 \pm 0.1 \mathrm{~cm}$
$L_{2}=38.7 \pm 0.1 \mathrm{~cm}$
$L_{3}=38.6 \pm 0.1 \mathrm{~cm}$
$L_{4}=38.5 \pm 0.1 \mathrm{~cm}$
a) What is the average or mean measurement?
[38.6 cm]
b) What is the range of measured values?
[0.2 cm]
c) What is the best stated value and its absolute uncertainty for the length of string?
$[\mathrm{L}=38.6 \pm 0.1 \mathrm{~cm}]$
4. Given two masses, $m_{1}=(100.0 \pm 0.4 \mathrm{~g})$ and $m_{2}=(49.3 \pm 0.3 \mathrm{~g})$, what is their sum, and what is their difference (both expressed with proper uncertainties)?
$[149.3 \pm 0.7 \mathrm{~g}, 50.7 \pm 0.7 \mathrm{~g}]$
5. With a good stopwatch and some practice, one can measure times ranging from about a second up to many minutes with an uncertainty of 0.2 second or so. Suppose that we wish to find the period T of a pendulum with $\mathrm{T}=0.5 \mathrm{~s}$. If we time one oscillation, we will have an uncertainty of about $40 \%$, but by timing several successive oscillations, we can do much better.
a) If we measure the time for five successive oscillations and get $2.4 \pm 0.2 \mathrm{~s}$, what is the final answer (with an absolute uncertainty) for the period?
[ $\mathrm{T}=0.48 \pm 0.04 \mathrm{~s}]$
b) What if we measure 20 oscillations and get a time of $9.4 \pm 0.2 \mathrm{~s}$ ?
$[\mathrm{T}=0.47 \pm 0.01 \mathrm{~s}]$
6. The digital stopwatch was started at a time $\mathrm{t}_{0}=0$ and then was used to measure ten swings of a simple pendulum to a time of $t=17.26 \mathrm{~s}$. If the time for ten swings of the pendulum is 17.26 s , what is the minimum absolute uncertainty in this measurement?
$[t=1.726 \pm 0.030 \mathrm{~s}]$
7. In an optical experiment a deflected ray of light is measured to be at an angle $\theta=(23 \pm 1)^{\circ}$ Find the sine of this angle and then determine the minimum and maximum acceptable values of this experimental value.
[ $0.39 \pm 0.02$ ]
8. Using $P=\frac{F}{A}$, work out P (pressure) from the experimental data to the right, $F=3.6 \pm 0.1 \mathrm{~N}$ with proper uncertainty.
$\left[70 \pm 20 \mathrm{Nm}^{-2}\right]$
$A=0.05 \pm 0.01 \mathrm{~m}^{2}$
9. Using $P=\frac{F}{A}$, work out P (pressure) from the experimental data to the right, with proper uncertainty.
[ $0.59 \pm 0.02 \mathrm{~Pa}$ ]
$F=66.4 \pm 0.1 \mathrm{~N}$
10. Using $P=\frac{V^{2}}{R}$,work out P (power) from the experimental data to the right, with proper uncertainty.
[2.8 $\pm 0.1 \mathrm{~W}]$
$V=10.0 \pm 0.1 \mathrm{~V}$
$R=36.0 \pm 0.5 \Omega$
11. Using $s=\frac{1}{2} a t^{2}$, work out s (distance) from the experimental data to the right, $a=5.0 \pm 0.1 \mathrm{~ms}^{-2}$ with proper uncertainty.
[26 $\pm 2 \mathrm{~m}$ ] $t=3.21 \pm 0.05 \mathrm{~s}$
12. Using $T=2 \pi \sqrt{\frac{l}{g}}$, work out $g$ (acceleration due to gravity) from the
13. Find the change of temperature from the following experimental results, with proper uncertainty:

Start temp $=30.2 \pm 0.1^{\circ} \mathrm{C}$
End temp $=36.4 \pm 0.1^{\circ} \mathrm{C}$
14. Using $v=u+a t$, work out $v$ from the experimental results from the right, $\quad u=5.0 \pm 0.1 \mathrm{~ms}-1$ with proper uncertainty.
[ $89 \pm 2 \mathrm{~ms}^{-1}$ ]
$a=2.00 \pm 0.02 \mathrm{~ms}^{-2}$
$t=42.0 \pm 0.5 \mathrm{~s}$
15. A computer interface is used to measure the position s (in cm ) of an object under uniform acceleration a (in $\mathrm{cms}^{-2}$ ) as a function of time t . The uncertainty in the time measurement is very small, about $\Delta t= \pm 0.0001 \mathrm{~s}$, and so you can ignore it, while the uncertainty in the distance is significant, where $\Delta s= \pm 0.1 \mathrm{~cm}$. Here is the data:

| Time <br> $t / s$ | $t^{2} / s^{2}$ | Distance <br> $s / \mathrm{cm}$ |
| :---: | :---: | :---: |
| 0.0000 | No data point | 0.0 |
| 0.0493 | 0.002430 | 0.1 |
| 0.1065 | 0.011342 | 0.5 |
| 0.1608 | 0.025857 | 1.1 |
| 0.1989 | 0.039561 | 1.7 |

The motion of the body can be described by the equation $s=\frac{1}{2} a t^{2}=0.5 t^{2}$. Therefore, graph of distance against the square of time should have a slope of 0.5 . Acceleration is determined by first finding the slope, where $a=2 \times$ slope .
a) Use the above data and graph distance against time squared on the grid provided, construct uncertainty bars for the distance points, and then determine the best straight line and solve for the acceleration. Do not include the origin in your data points.

b) On your graph in part (a), draw the minimum and maximum slopes on graph and determine the range of acceleration based on these two extreme slope values.

Use graphing software to work questions 15-23, but remember that you should to be able to do any of them by hand on graph paper.
16. The data table shows a relationship between $x$ and $y$.
a) Graph the data.
b) Knowing that the data fits a cubic relationship $\left(x^{3}\right)$, transform the data.
c) Make an appropriate graph of the transformed data and determine the equation of the function.

| $\mathbf{x}$ | $\mathbf{y}$ |
| :---: | :---: |
| 0 | 0 |
| 0.2 | 0.03 |
| 0.4 | 0.26 |
| 0.6 | 0.86 |
| 0.8 | 2.05 |
| 1.0 | 4.00 |
| 1.2 | 6.91 |
| 1.4 | 10.98 |

17. The data table shows a relationship between $x$ and $y$.
a) Graph the data.
b) What type of curve do you think this is?
c) What is your best guess equation for this function?

| $\mathbf{x}$ | $\mathbf{y}$ |
| :---: | :---: |
| 0 | 1.0 |
| 0.5 | 1.4 |
| 1.0 | 2.0 |
| 1.5 | 2.8 |
| 2.0 | 4.0 |
| 2.5 | 5.7 |
| 3.0 | 8.0 |

18. The table below shows the force needed to extend a spring by a given distance (Hooke's Law). The relationship between the two is linear.
a) Graph the data.
b) Add error bars to your graph.
c) Add a line of best fit.
d) Find the slope of your line.
e) Find the $y$-intercept.
f) Write the equation for your line in the form $y=m x+b$.
g) Add max and min lines to your graph.
h) What is the range of possible slopes within your error bars?

| Length (cm) <br> $\mathbf{+} / \mathbf{0 . 2} \mathbf{~ c m}$ | Force $\mathbf{( N )}$ <br> $\mathbf{+} \mathbf{- 0 . 5} \mathbf{~ N}$ |
| :---: | :---: |
| 1.0 | 5.0 |
| 2.0 | 8.0 |
| 3.0 | 11.0 |
| 4.0 | 14.0 |
| 5.0 | 17.0 |
| 6.0 | 20.0 |
| 7.0 | 23.0 |
| 8.0 | 26.0 |
| 9.0 | 29.0 |
| 10.0 | 32.0 |

19. The table shows experimental distance and time data for a falling object near the Earth's surface (constant acceleration).
a) Graph the data and add error bars to your graph.
b) Draw in a curve that fits the data.
c) Make a table of $x^{2}$ and $y$.
d) Graph $y$ in terms of $x^{2}$ and draw in a line of best fit.
e) Find the slope of your line.
f) Write the function in the form $y=a x^{2}$.

| Time $(\mathbf{s})$ |  |
| :---: | :---: |
| $\mathbf{+} / \mathbf{- 0 . 2 ~ s}$ | Distance $(\mathbf{m})$ <br> $\mathbf{+} / \mathbf{0 . 1} \mathbf{~ m}$ |
| 1.0 | 0.5 |
| 2.0 | 2.0 |
| 3.0 | 4.5 |
| 4.0 | 8.0 |
| 5.0 | 12.5 |
| 6.0 | 18.0 |

20. The table shows experimental data relating the length of a pendulum to its period.
a) Plot a graph of $y$ as a function of $x$ with error bars.
b) Describe its shape and draw in a curve that fits the data.
c) The relation between $x$ and $y$ is thought to be $y=a(x)^{0.5}$. Transform your data so that you will be able to graph $y$ as a function of $x^{0.5}$ and then draw the graph, with error bars.
d) From your graph, determine the constant a, with

| Length ( $\mathbf{0 . 1} \mathbf{~ c m}$ ) | Period ( $\mathbf{0} \mathbf{0 . 3 ~ s}$ ) |
| :---: | :---: |
| 1.0 | 2.0 |
| 2.0 | 2.8 |
| 4.0 | 4.0 |
| 6.0 | 4.9 |
| 7.0 | 5.3 |
| 9.0 | 6.0 |
| 16.0 | 8.0 |

21. In an experiment, two variable quantities, $x$ and $y$ were measured. The relation between $x$ and $y$ is thought to be: $\mathbf{y}=\mathbf{a x} \mathbf{x}^{1 / 2}$.

| $\mathbf{x}( \pm \mathbf{0 . 0 1} \mathbf{~ m})$ | $\mathbf{y}( \pm \mathbf{0 . 0 5} \mathbf{s})$ |
| :---: | :---: |
| 0.10 | 0.64 |
| 0.20 | 0.90 |
| 0.40 | 1.27 |
| 0.80 | 1.80 |
| 1.00 | 1.90 |

a) Plot a graph which could be used to verify the relation between $x$ and $y$.
b) Find the value of a, reported with appropriate uncertainty (you will need a best fit line, error bars and $\mathrm{max} / \mathrm{min}$ lines).
22. In an experiment, two variable quantities $x$ and $y$ were measured. The relation between $x$ and $y$ is thought to be: $y=b x^{3}$
a) Plot a graph which could be used to verify the relation

| $\mathbf{x}( \pm \mathbf{0 . 0 2} \mathbf{~ c m})$ | $\mathbf{y}(\mathbf{0 . 3 0} \mathbf{~ g})$ |
| :---: | :---: |
| 0.67 | 2.25 |
| 0.80 | 4.00 |
| 0.90 | 5.52 |
| 0.95 | 6.60 | between x and y .

b) Find the value of $\mathbf{b}$, reported with appropriate uncertainty (you will need a best fit line, error bars and max/min lines).
c) What physical quantity might $\mathbf{b}$ represent?
23. A point source of light was placed at different distances (r) from a photoelectric cell. The current, I, generated by the cell was measured. It is thought that the relation between $I$ and $r$ is of the form: $\boldsymbol{I}=\mathbf{k r}^{\mathbf{n}}$ where $k$ and n are constants. Taking logs of both sides of this equation gives us: log $\mathbf{l}=\boldsymbol{\operatorname { l o g }} \mathbf{k}+\mathbf{n} \log \mathbf{r}$.
a) Plot a graph of $\log r$ (horizontal) against log I .
b) Find the values of the constants k and n , reported with appropriate uncertainty (you will need a best fit line, error bars and max/min lines).
c) A significant error was made in one of the measurements. Identify the measurement and suggest a possible corrected value for this result.

| $\mathbf{r}$ <br> $\mathbf{( \pm 0 . 5} \mathbf{~ c m})$ | $\mathbf{I}$ <br> $( \pm \mathbf{2} \mathbf{~ m A})$ |
| :---: | :---: |
| 15.0 | 108 |
| 20.0 | 61 |
| 25.0 | 39 |
| 30.0 | 27 |
| 35.0 | 24 |
| 40.0 | 15 |
| 45.0 | 12 |

