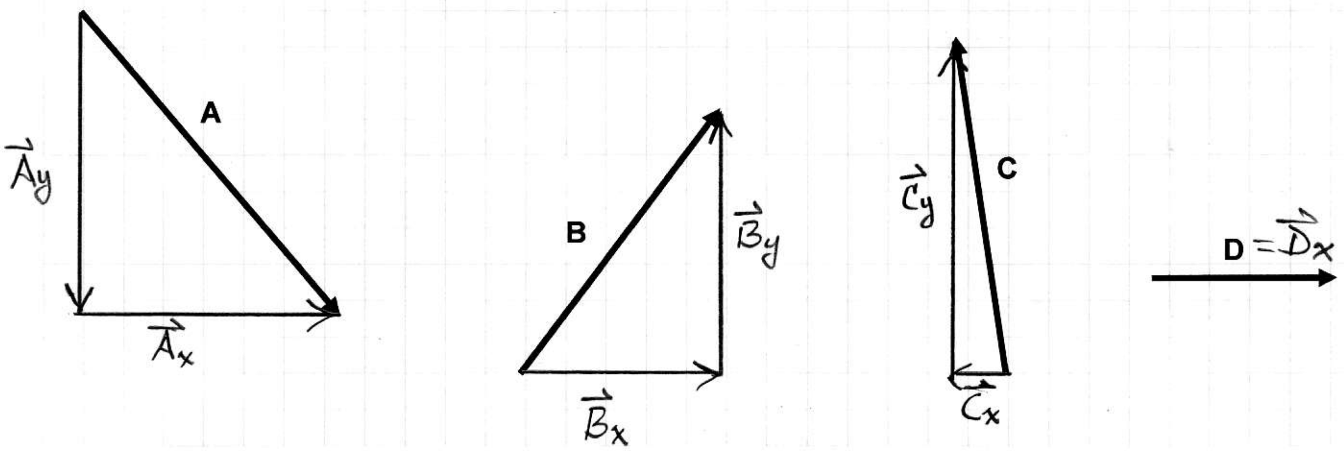
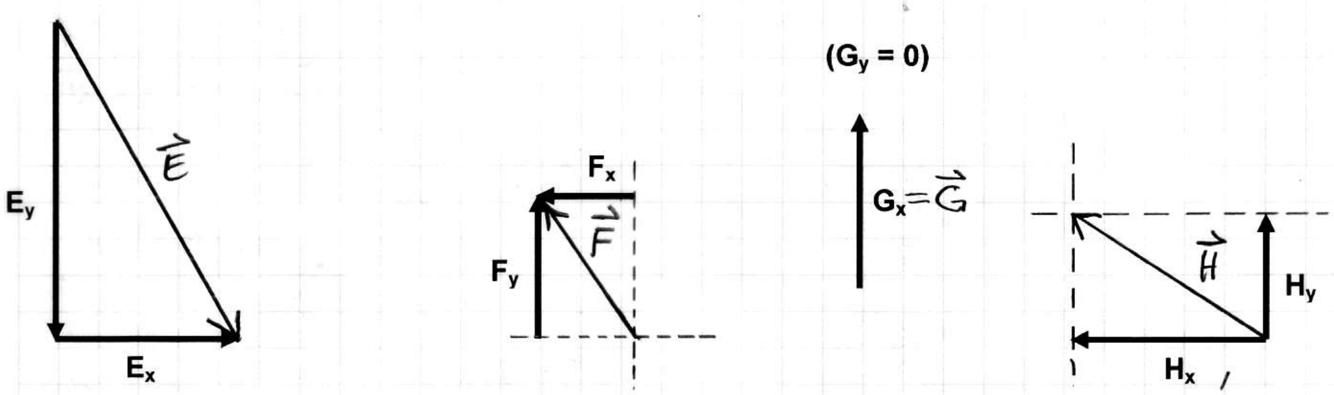
1.3 VECTORS AND SCALARS — *HW Solutions*

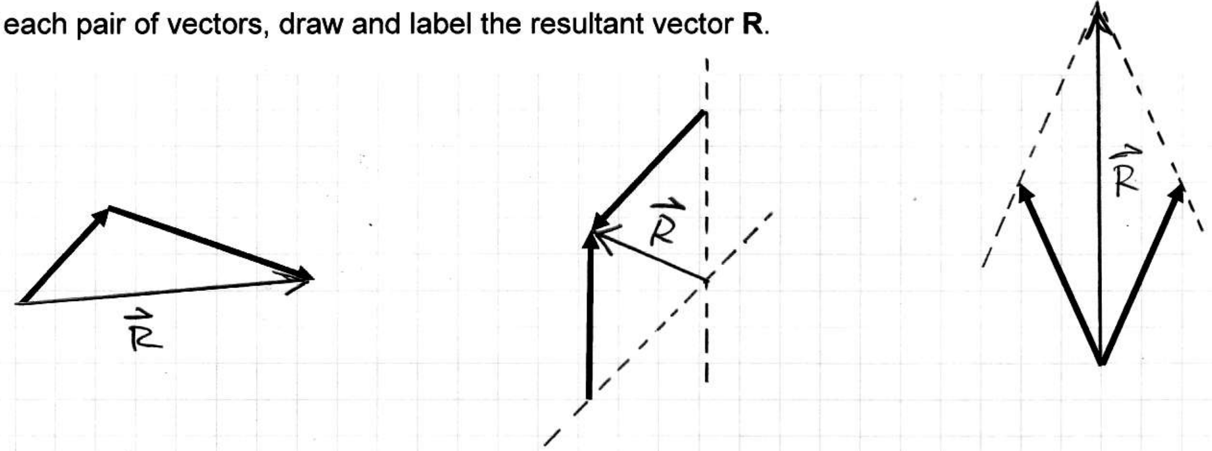
1. For each vector, draw and label its x and y components:



2. For each set of component vectors, draw and label the resultant vector.



3. For each pair of vectors, draw and label the resultant vector **R**.



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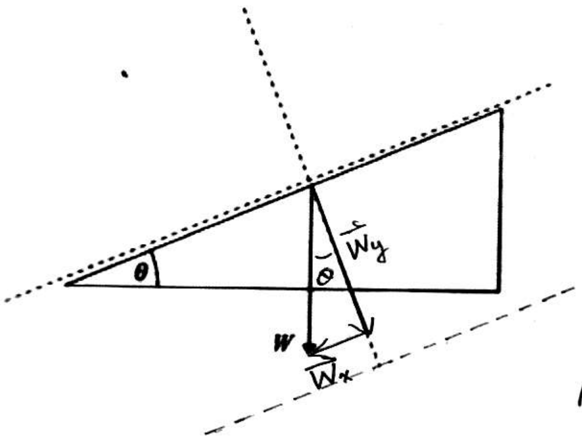
Phet **4.** The components of a vector are given as $\mathbf{s}_x$ = 17.0 m and $\mathbf{s}_y$ = 8.0 m. Determine the resultant vector **s** graphically and analytically.

Phet **5.** The components of a vector are given as $\mathbf{v}_x$ = - 5.0 ms^{-1} and $\mathbf{v}_y$ = 20.0 ms^{-1}. Determine the resultant vector **v** graphically and analytically.

Phet **6.** **B** is a vector 15.8 units in magnitude at 34.7° above the –x axis
- a) Sketch this vector, choosing an appropriate scale.
- b) Find $\mathbf{B}_x$ and $\mathbf{B}_y$ by measuring with a ruler.
- c) Find $\mathbf{B}_x$ and $\mathbf{B}_y$ analytically.
- d) Use $\mathbf{B}_x$ and $\mathbf{B}_y$ from parts b) and c) to obtain (again) the magnitude and direction of **B**. Are they the same? Why or why not?

7. An eagle leaves his nest and flies 22.0 km north. He then flies in a direction 60.0° south of east for 47.0 km. What is the eagle's displacement from his nest?

8. Draw and compute the components of the vector **w** as shown, along the axes indicated, if **w** = 12.5 N and θ = 23.0°.



FOR THE SMALL Δ:

$\sin\theta = \frac{W_x}{W} \Rightarrow W_x = W\sin\theta$

$= (12.5)(\sin 23.0)$

$\boxed{W_x = {}^{-}4.88\ N}$

$\cos\theta = \frac{W_y}{W} \Rightarrow W_y = W\cos\theta$

$= (12.5)(\cos 23.0)$

$\boxed{W_y = {}^{-}11.5\ N}$

NOTE: W_x is ∥ to the inclined plane.
W_y is ⊥ to the inclined plane.

SEE ATTACHED PAGE

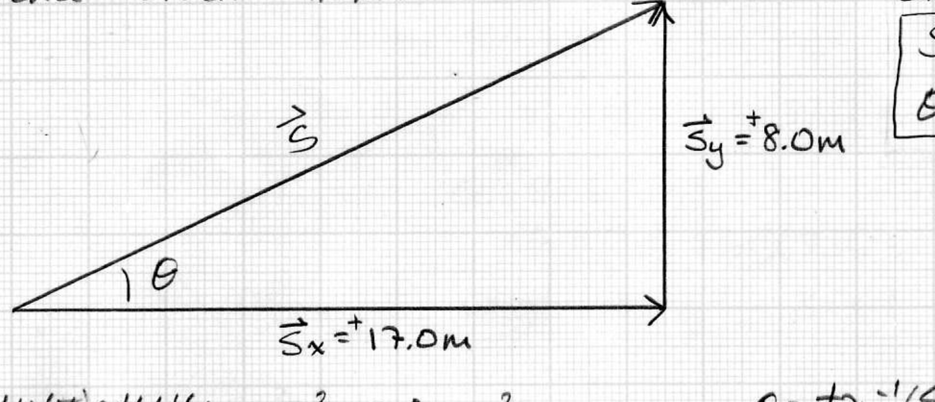
9. An airplane is traveling at **v** = 735 km/h in a direction 42.5° west of north.
- a) Find the components of **v** in the northerly and westerly directions.
- b) How far north and how far west has the plane traveled in 3.00 h?

497
542
42.5
sin 42.5

Phet **10.** Vector **D** is 7.0 cm long at 180° and vector **E** is 8.5 cm long at 45°.
- a) Find the x and y components of **D** and **E**.
- b) Determine **D** + **E** (magnitude and direction).

11. An airplane trip involves three legs, with two stopovers. The first leg is due east (90.0°) for 620 km; the second leg is southeast (315.0°) for 440 km and the third leg is 53.0° south of west, for 550 km. What is the plane's total displacement?

④ SCALE: 0.5 cm = 1.0 m



GRAPHICALLY:

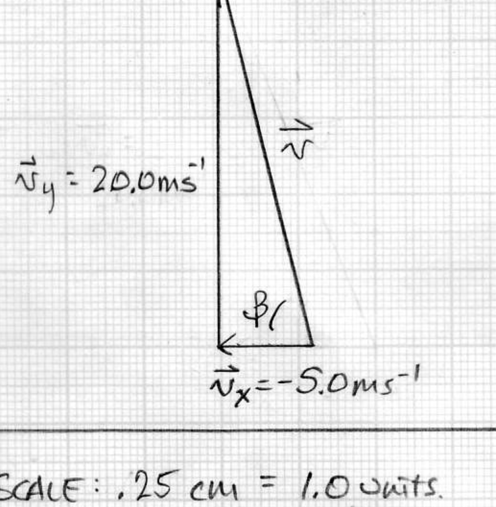
$\boxed{S = 18.2\,m \quad \theta = 25°}$

ANALYTICALLY:

$$S^2 = S_x^2 + S_y^2$$
$$S = (S_x^2 + S_y^2)^{\frac{1}{2}}$$
$$= ((17.0)^2 + (8.0)^2)^{1/2}$$
$$\boxed{S = 19\ m}$$

$$\theta = \tan^{-1}\left(\frac{S_y}{S_x}\right)$$
$$= \tan^{-1}\left(\frac{8.0}{17.0}\right)$$
$$\boxed{\theta = 25°}$$

⑤ SCALE: 1 cm = 4 ms^{-1}



GRAPHICALLY:

$\boxed{v = 21\ ms^{-1} \quad \theta = 75°}$

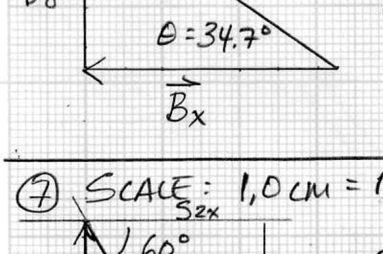
ANALYTICALLY:

$$v = (v_x^2 + v_y^2)^{1/2} = ((5.0)^2 + (20.0)^2)^{1/2}$$
$$v = 20.61$$
$$\boxed{v = 21\ ms^{-1}}$$

$$\phi = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{20.0}{5.0}\right) \Rightarrow \boxed{\theta = 76°}$$

⑥ SCALE: .25 cm = 1.0 units.

a)



b) $\boxed{\vec{B}_x = -13.1 \text{ units} \quad \vec{B}_y = 9 \text{ units.}}$

c) $\sin(34.7) = \frac{B_y}{15.8}$

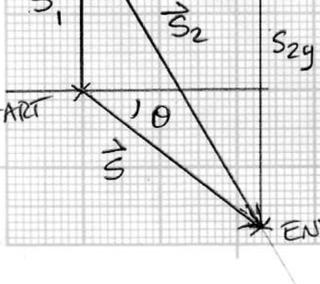
$\boxed{B_y = 8.99 \text{ units.}}$

$\cos(34.7) = \frac{B_x}{15.8}$

$\boxed{B_x = 13.0 \text{ units}}$

d) Close but not exactly because there is likely some error in measurement.

⑦ SCALE: 1.0 cm = 10 km.



$$\sin 60° = \frac{S_{2y}}{47.0}$$

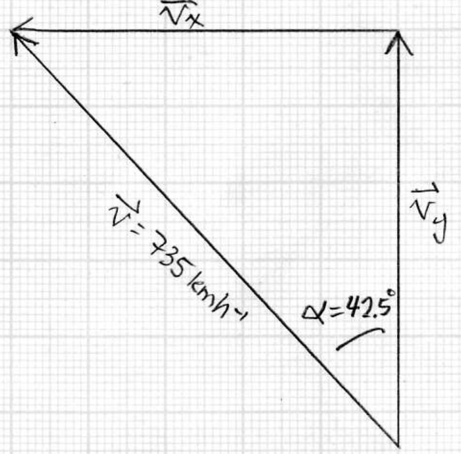
$$\cos 60° = \frac{S_{2x}}{47.0}$$

	x	y
S_1	0	+22.0
S_2	23.5	−40.7
S	+23.5	−18.7

$$S = (S_x^2 + S_y^2)^{1/2}$$
$$= (23.5^2 + -18.7^2)^{1/2}$$
$$\boxed{S = 30.0\ km}$$

$$\theta = \tan^{-1}\left(\frac{S_y}{S_x}\right)$$
$$= \tan^{-1}\left(\frac{18.7}{23.5}\right)$$
$$\boxed{\theta = 38.5°}$$

⑨ SCALE: 1.0 cm = 100 kmh^{-1}



a) $\sin 42.5 = \frac{V_x}{735}$

$\Rightarrow V_x = (\sin 42.5)(735) \Rightarrow \boxed{V_x = {}^{-}497\ kmh^{-1}}$

$\cos 42.5 = \frac{V_y}{735}$

$V_y = (\cos 42.5)(735) \Rightarrow \boxed{V_y = {}^{+}542\ kmh^{-1}}$

b) WEST : $v = \frac{s}{t} \Rightarrow s = vt = (497)(3.00)$
$= \boxed{1490\ km}$

NORTH : $v = \frac{s}{t} \Rightarrow s = vt = (542)(3.00)$
$= \boxed{1630\ km}$

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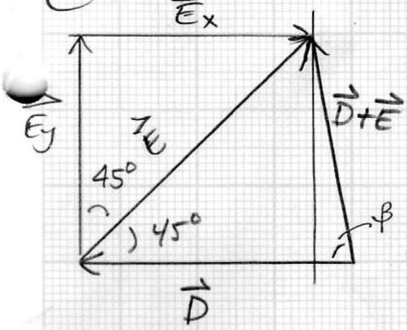
⑩ SCALE: 0.5 cm = 1.0 cm.



a) $\boxed{\begin{array}{l} D_x = -7.0\,cm \\ D_y = 0\,cm. \end{array}}$

$\sin 45 = \frac{E_x}{8.5}$

$\boxed{E_x = {}^{+}6.0\,cm = E_y}$

b)

	x	y
D	−7.0	0
E	+6.0	+6.0
R	−1.0	+6.0.

$R = (R_x{}^2 + R_y{}^2)^{1/2} = ((-1.0)^2 + (+6.0)^2)^{1/2}$

$\boxed{R = 6.1\,cm}$

$\beta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\left(\frac{6}{1}\right)$

$\boxed{\beta = 81°}$

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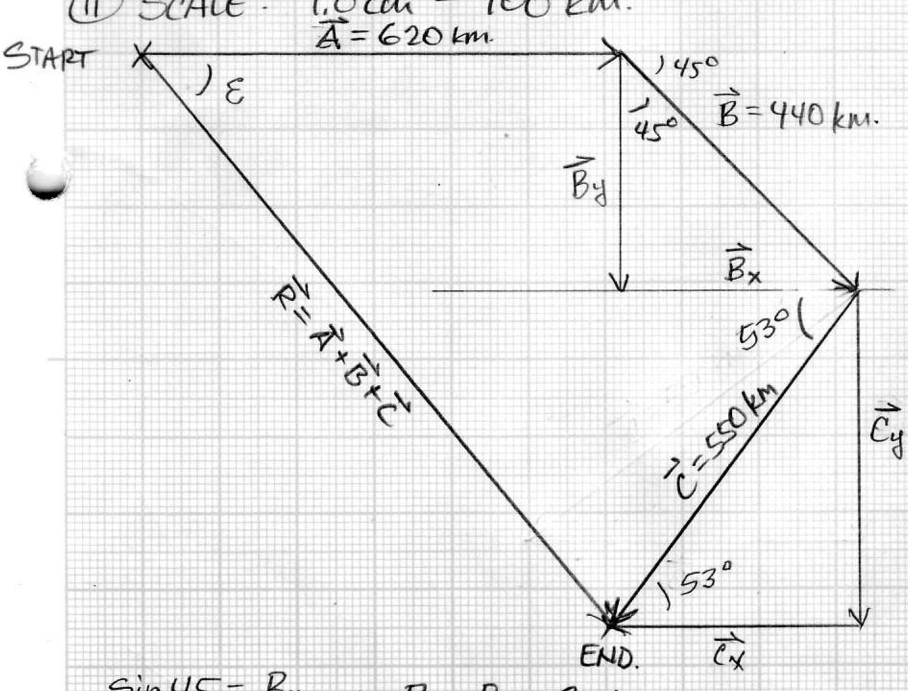
⑪ SCALE: 1.0 cm = 100 km.



	x	y
A	+620	0
B	+311	−311
C	−330	−440
R	601	−751

$R = (R_x{}^2 + R_y{}^2)^{1/2}$
$= (601^2 + 751^2)^{1/2}$

$\boxed{R = 962\ km}$

$\varepsilon = \tan^{-1}\left(\frac{R_y}{R_x}\right)$

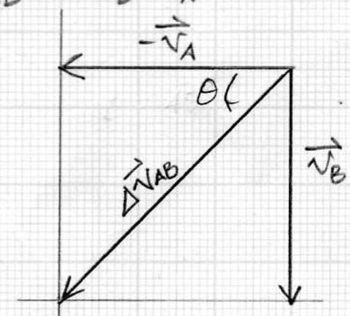
$\boxed{\varepsilon = 51.3°}$

$\sin 45 = \frac{B_x}{440} \Rightarrow B_x = B_y = \underline{311\ km.}$

$\sin 53 = \frac{C_y}{550} \Rightarrow C_y = \underline{440\ km}$

$\cos 53 = \frac{C_x}{550} \Rightarrow C_x = \underline{330\ km}$

(12) a) $\Delta\vec{v}_{AB} = \vec{v}_B - \vec{v}_A = \vec{v}_B + -\vec{v}_A$ LOOKS LIKE:

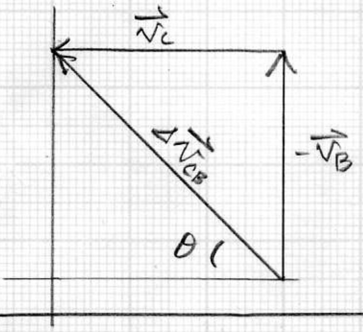


$v_{AB} = (v_A^2 + v_B^2)^{1/2}$

$= (6.0^2 + 6.0^2)^{1/2} \Rightarrow$ $\boxed{v_{AB} = 8.5\ ms^{-1} \quad \theta = 45°}$

- $\vec{v}$ CONSTANTLY CHANGES BECAUSE IT'S DIRECTION IS CONSTANTLY CHANGING.
- v DOES NOT CHANGE.
- $\Delta\vec{v}$ POINTS TOWARD THE CENTER OF THE CIRCLE!

b) $\Delta\vec{v}_{BC} = \vec{v}_C - \vec{v}_B = \vec{v}_C + -\vec{v}_B$ LOOKS LIKE:



$v_{CB} = (v_C^2 + v_B^2)^{1/2}$

$= (6.0^2 + 6.0^2)^{1/2} \Rightarrow$ $\boxed{v_{BC} = 8.5\ ms^{-1} \quad \theta = 45°}$

- Δv POINTS TOWARD THE CENTER OF THE CIRCLE AS BEFORE!

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(13)

VECTOR A:

$\sin 28.0 = \frac{A_y}{44.0} \Rightarrow A_y = {}^+20.1$

$\cos 28.0 = \frac{A_x}{44.0} \Rightarrow A_x = {}^+38.8$

VECTOR B:

$\sin 56.0 = \frac{B_y}{26.5} \Rightarrow B_y = {}^+22.0$

$\cos 56.0 = \frac{B_x}{26.5} \Rightarrow B_x = -14.8$

VECTOR C: $C_x = 0;\ C_y = C$

	x	y
A	+38.8	+20.1
B	-14.8	+22.0
C	0	-31.0
R	+24.0	+11.1

a) $\vec{R} = \vec{A} + \vec{B} + \vec{C}$: $R = (R_x^2 + R_y^2)^{1/2} = (24.0^2 + 11.1^2)^{1/2} = \boxed{26.4 \text{ units} = R}$

$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{11.1}{24.0}\right) \Rightarrow \boxed{\theta = 24.8}$

VERIFY GRAPHICALLY!

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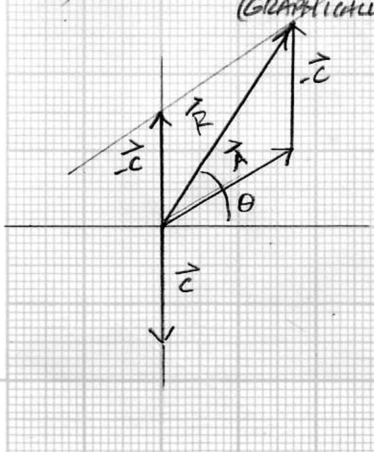
b) $\vec{A} - \vec{C} = \underset{\text{(GRAPHICALLY)}}{\vec{A} + -\vec{C}} = (A_x + -C_x)\hat{i} + (A_y + -C_y)\hat{j} = (38.8 + 0)\hat{i} + (20.1 + 31.0)\hat{j}$



OR $\vec{R} = 38.8\hat{i} + 51.1\hat{j} = \vec{R}_x + \vec{R}_y$

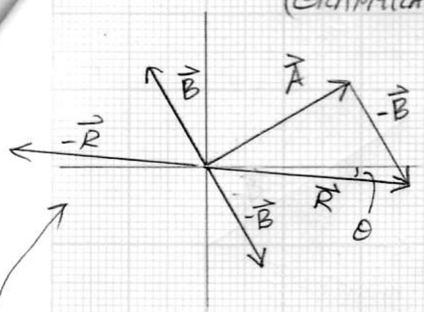
THEREFORE, $R = (R_x^2 + R_y^2)^{1/2}$

$= (38.8^2 + 51.1^2)^{1/2} \Rightarrow \boxed{R = 64.2 \text{ units}}$

$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{51.1}{38.8}\right) \Rightarrow \boxed{\theta = 52.8}$

c) $\vec{A}-\vec{B} = \vec{A} + -\vec{B} = (A_x - B_x)\hat{i} + (A_y - B_y)\hat{j}$

(GRAPHICALLY)



$= (38.8 - -14.8)\hat{i} + (20.1 - 22.0)\hat{j}$

$= 53.6\hat{i} + -1.9\hat{j}$

$= \vec{R}_x + \vec{R}_y$

So, $R = (R_x^2 + R_y^2)^{1/2} = (53.6^2 + 1.9^2)^{1/2}$

$\boxed{R = 53.6 \text{ units.}}$

$\theta = \tan^{-1}\left(\frac{1.9}{53.6}\right) \Rightarrow \boxed{\theta = 2.0° \text{ BELOW } +x \text{ AXIS.}}$

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d) $\vec{B}-\vec{A} = -(\vec{A}-\vec{B}) = -\vec{R}$ FROM PREVIOUS PROBLEM.

$\Rightarrow \boxed{R = 53.6 \text{ units}, \ \theta = 2.0° \text{ ABOVE } -x \text{ AXIS.}}$

GRAPHICALLY, SEE DIAGRAM IN (c).

$\vec{B}-\vec{A} = \vec{B} + -\vec{A} = (B_x - A_x)\hat{i} + (B_y - A_y)\hat{j}$

$= (-14.8 - 38.8)\hat{i} + (22.0 - 20.1)\hat{j}$

$= -53.6\hat{i} + 1.9\hat{j}$

$= \vec{R}_x + \vec{R}_y$

So, $\vec{R} = (R_x^2 + R_y^2)^{1/2} = (53.6^2 + 1.9^2)^{1/2}$

$\boxed{R = 53.6 \text{ units.}}$

$\theta = \tan^{-1}\left(\frac{1.9}{53.6}\right) \Rightarrow \boxed{\theta = 2.0°}$ NOTE: DRAWING IS CRUCIAL HERE FOR PROPER DIRECTION.

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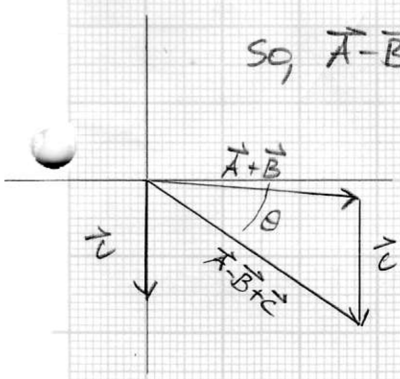
e) $\vec{A}-\vec{B}+\vec{C}$ WE ALREADY FOUND $\vec{A}-\vec{B}$ IN PART c). IT WAS:

$\vec{A}-\vec{B} = 53.6\hat{i} + -1.9\hat{j}$

So, $\vec{A}-\vec{B}+\vec{C} = (53.6 + 0)\hat{i} + (-1.9 + -31.0)\hat{j}$

$= 53.6\hat{i} + -32.9\hat{j}$

$= \vec{R}_x + \vec{R}_y$

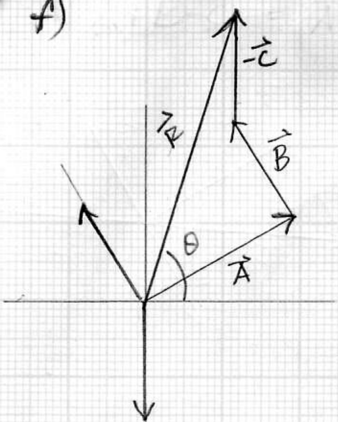


So, $R = (R_x^2 + R_y^2)^{1/2}$

$= (53.6^2 + 32.9^2)^{1/2}$

$\boxed{R = 62.9 \text{ units.}}$

$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{32.9}{53.6}\right) \Rightarrow \boxed{\theta = 31.5° \text{ BELOW } +x \text{ AXIS.}}$

f) 

$\vec{A}+\vec{B}-\vec{C} = \vec{A}+\vec{B}+-\vec{C}$ (GRAPHICALLY)

$= (A_x + B_x - C_x)\hat{i} + (A_y + B_y - C_y)\hat{j}$

$= (38.8 - 14.8 - 0)\hat{i} + (20.1 + 22.0 - -31.0)\hat{j}$

$= 24.0\hat{i} + 73.1\hat{j}$

$= R_x + R_y$

SO, $R = (R_x{}^2 + R_y{}^2)^{1/2} = (24.0^2 + 73.1^2)^{1/2}$

$\boxed{R = 76.9 \text{ units.}}$

$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{73.1}{24.0}\right) \Rightarrow \boxed{\theta = 71.8°}$

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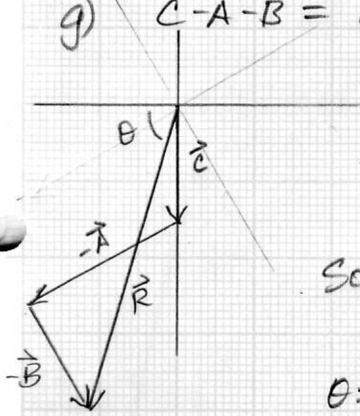
g) $\vec{C}-\vec{A}-\vec{B} = \vec{C}+-\vec{A}+-\vec{B}$ (GRAPHICALLY)



$= (C_x - A_x - B_x)\hat{i} + (C_y - A_y - B_y)\hat{j}$

$= (0 - 38.8 - -14.8)\hat{i} + (-31.0 - 20.1 - 22.0)\hat{j}$

$= -24.0\hat{i} + -73.1\hat{j}$

$= \vec{R}_x + \vec{R}_y$

So, $R = (R_x{}^2 + R_y{}^2)^{1/2} = (-24.0^2 + -73.1^2)^{1/2}$

$\boxed{R = 76.9 \text{ units.}}$

$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{73.1}{24.0}\right) \Rightarrow \boxed{\theta = 71.8° \text{ BELOW } -x \text{ AXIS.}}$

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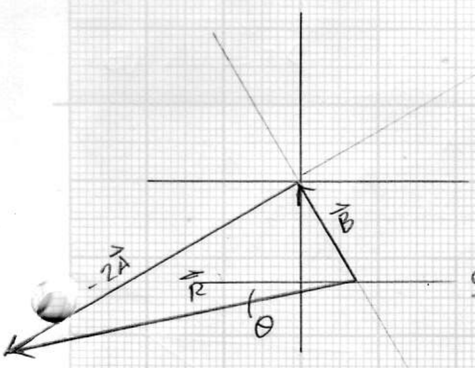
h) $\vec{B}-2\vec{A} = \vec{B}+-2\vec{A}$ GRAPHICALLY.



$= (B_x - 2A_x)\hat{i} + (B_y - 2A_y)\hat{j}$

$= [-14.8 - 2(38.8)]\hat{i} + [22.0 - 2(20.1)]\hat{j}$

$= -92.4\hat{i} + -18.2\hat{j}$

$= \vec{R}_x + \vec{R}_y$

SO, $R = (R_x{}^2 + R_y{}^2)^{1/2} = (92.4^2 + 18.2^2)^{1/2}$

$\boxed{R = 94.2 \text{ units}}$

$\theta = \tan^{-1}\left(\frac{18.2}{92.4}\right)$

$\boxed{\theta = 11.1° \text{ BELOW } -x \text{ AXIS.}}$

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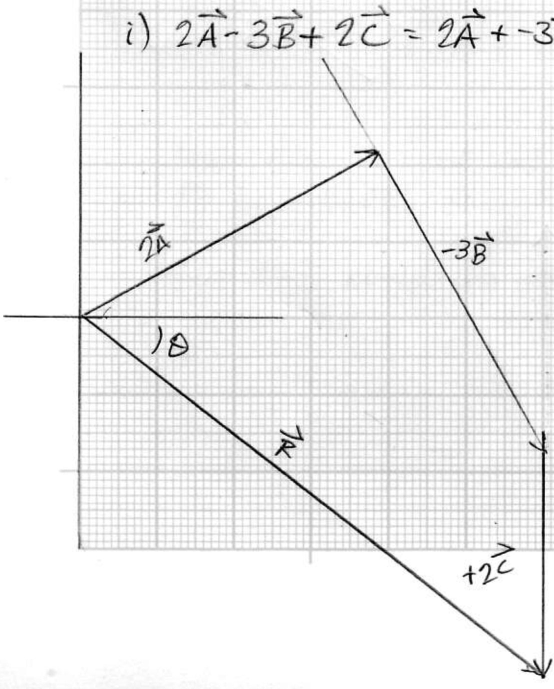
i) $2\vec{A}-3\vec{B}+2\vec{C} = 2\vec{A}+-3\vec{B}+-2\vec{C}$ (GRAPHICALLY)



$= (2A_x - 3B_x + 2C_x)\hat{i} + (2A_y - 3B_y + 2C_y)\hat{j}$

$= [2(38.8) - 3(-14.8) + 2(0)]\hat{i} + [(2(20.1) - 3(22.0) + 2(-31.0)]\hat{j}$

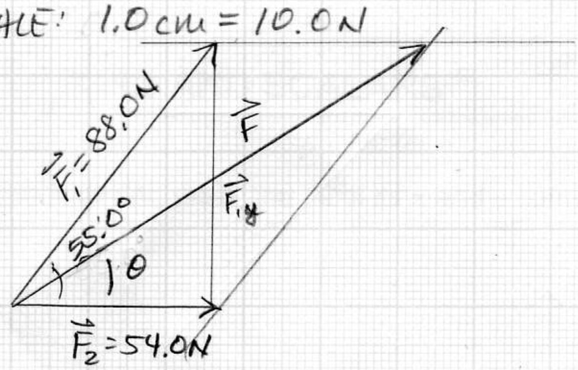
$= 122\hat{i} + -87.8\hat{j}$

$= \vec{R}_x + \vec{R}_y$

So, $R = (R_x{}^2 + R_y{}^2)^{1/2} = (122^2 + 87.8^2)^{1/2} \Rightarrow \boxed{R = 150. \text{ units.}}$

$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{87.8}{122}\right) \Rightarrow \boxed{\theta = 35.7° \text{ BELOW } +x \text{ AXIS}}$

(14) SCALE: 1.0 cm = 10.0 N



	x	y
F_1	+50.5	+72.0
F_2	+54.0	0
F	+104.5	+72.0.

$F = (F_x^2 + F_y^2)^{1/2} = (104.5^2 + 72.0^2)^{1/2}$

$\boxed{F = 127\ N}$.

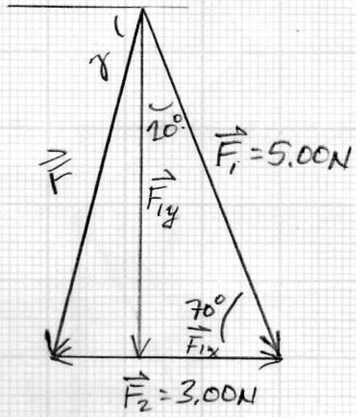
$\sin 55.0 = \frac{F_{1y}}{88.0} \Rightarrow F_{1y} = 72.0N$

$\cos 55.0 = \frac{F_{1x}}{88.0} \Rightarrow F_{1x} = 50.5N$

$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$

$= \tan^{-1}\left(\frac{72.0}{104.5}\right) \Rightarrow \boxed{\theta = 34.6°}$.

(15) SCALE: 1.0 cm = 1.0 N



$\sin 70 = \frac{F_{1y}}{5.00}$

$F_{1y} = 4.70N$

$\cos 70 = \frac{F_{1x}}{5.00}$

$F_{1x} = 1.71N$

	x	y
F_1	+1.71	-4.70.
F_2	-3.00	0
F	-1.29	-4.70.

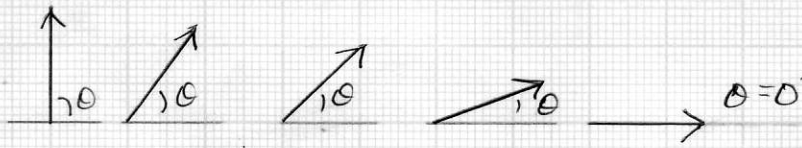
$F = (F_x^2 + F_y^2)^{1/2} = (1.29^2 + 4.70^2)^{1/2}$

$\boxed{F = 4.87\ N}$

$\gamma = \tan^{-1}\left(\frac{F_y}{F_x}\right) \Rightarrow \boxed{\gamma = 74.7°}$

THE BOX MOVES TO THE LEFT.

(16)



- at θ = 90°, $F_x = 0$ and $F_y = F$. HENCE F_y IS AT MAXIMUM AND F_x IS AT MINIMUM.
- at θ = 0°, $F_x = F$ and $F_y = 0$. HENCE F_x IS AT MAXIMUM AND F_y IS AT MINIMUM.

FOR EACH VALUE IN BETWEEN, YOU CAN SEE THAT AS θ DECREASES, F_y DECREASES AND F_x INCREASES.

BUT THE MAXIMUM VALUES OF F_x AND F_y WILL NEVER EXCEED F!