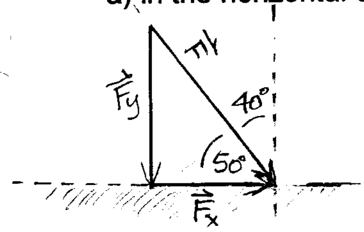
2.2 FORCES AND DYNAMICS — *HW/Study Packet Solutions*

1. A man pushes a lawnmower with a force of 80 N directed along the handle which makes an angle of 40° to the vertical. What is the magnitude of the force:

a) in the horizontal direction? b) in the vertical direction?



$\cos\theta = \frac{F_x}{F}$

$F_x = F\cos\theta$

$= 80 \cdot \cos 50°$

$F_x = 51.42\,N = \boxed{^{+}50N}$

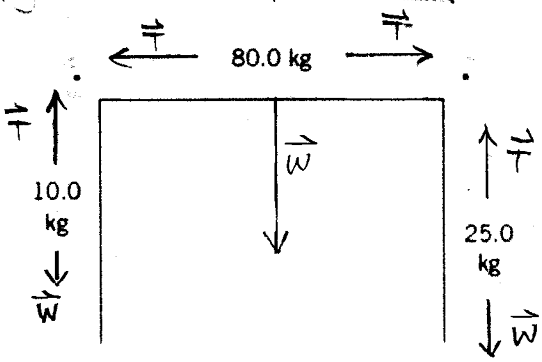
$\sin\theta = \frac{F_y}{F}$

$F_y = F\sin\theta$

$= 80 \cdot \sin 50°$

$F_y = 61.28\,N = \boxed{60N}$

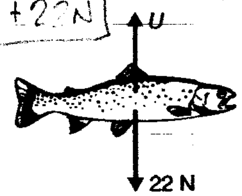
2. On the diagram to the right, draw vectors showing all forces.



3. The body shown in each of the following free-body diagrams is in equilibrium. Write down the value(s) of the unknown force(s) in each case.

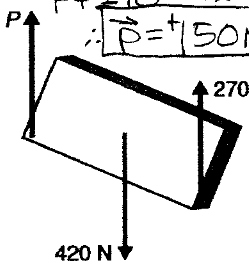
$\vec{U} + {}^{-}22 = 0$

$\therefore \boxed{\vec{U} = +22N}$

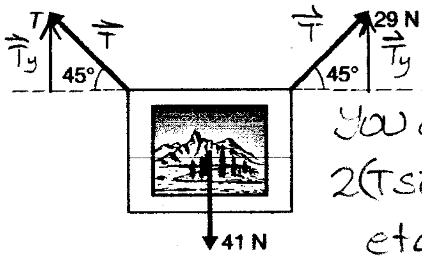


$\vec{P} + 270 - 420 = 0$

$\therefore \boxed{\vec{P} = {}^{+}150N}$



$\vec{T}$ must be $\boxed{+29N}$ by symmetry.



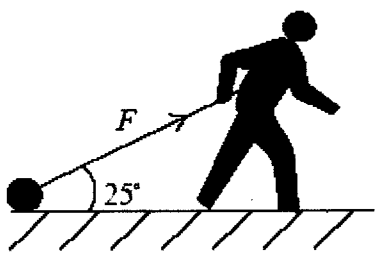
YOU CAN VERIFY:

$2(T\sin 45) - 41 = 0$

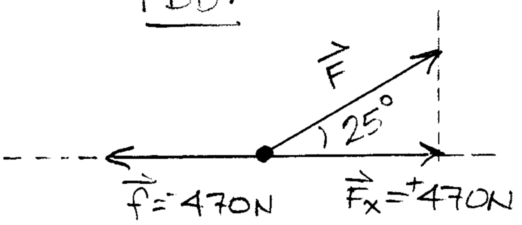
etc...

4. An athlete trains by dragging a heavy load across a rough horizontal surface as shown. The athlete exerts a force of magnitude *F* on the load at an angle of 25° to the horizontal. Once the load is moving at a steady speed the average horizontal frictional force *f* acting on the load is 470 N. Calculate the average value of *F* that will enable the load to move at constant speed.

(EQUILIBRIUM)



FBD:



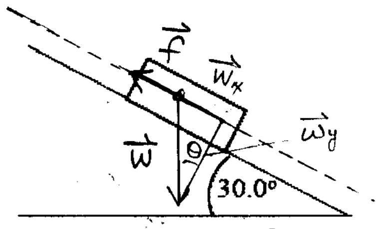
$f = F_x$ IF IN EQUILIBRIUM (NOT ACCELERATING).

$\cos\theta = \frac{F_x}{F} \Rightarrow F = \frac{F_x}{\cos\theta} = \frac{470}{\cos 25}$

$= 518.6\,N$

$\boxed{F = 520N}$

5. A block is resting on a rough slope (and not moving) as shown.



$\theta = 30.0°$

a) Label the diagram showing all the forces acting on the block.

b) If the block has a mass of 4 kg and is stationary what is the friction force between the block and the surface?

IF STATIONARY, THE BLOCK IS IN EQUILIBRIUM AND $f = w_x$

$\sin\theta = \frac{w_x}{w} \Rightarrow w_x = w\sin\theta$

$= mg\sin\theta$

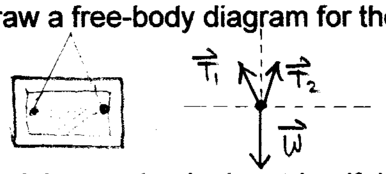
$= (4)(9.81)(\sin 30)$

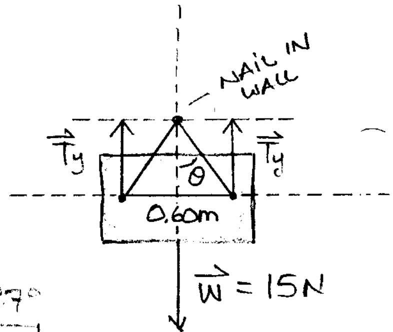
$\vec{w}_x = 19.62\ N = \boxed{20N = f}$

Note: $\vec{w}_x$ and $\vec{w}_y$ are not real vectors, they are the components of $\vec{w}$.

6. A framed picture of weight 15 N is to be hung on a wall, using a piece of string. The ends of the string are tied to two points, 0.60 m apart on the same horizontal level, on the back of the picture.

a) Draw a free-body diagram for the picture.





b) find the tension in the string if the string is 1.0 m long.

IF 1.0M LONG THEN EACH SIDE IS 0.5M LONG, AND

$\sin\theta = \frac{0.30}{0.50}$ SO, $\cos\theta = \frac{T_y}{T} \Rightarrow T = \frac{T_y}{\cos\theta} = \frac{7.5}{\cos 37°}$

$\theta = 37°$

$\boxed{T = 9.4N}$

$2T_y = 15N$

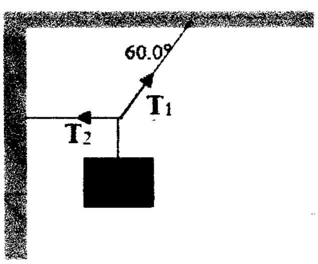
$T_y = 7.5N$

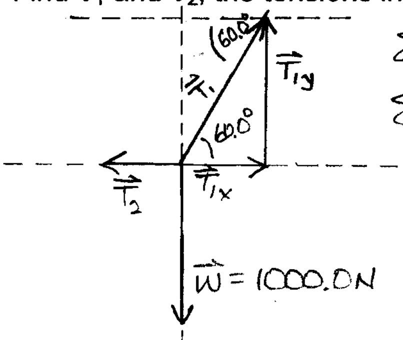
c) find the tension in the string if the string is 0.66 m long.

NOW, θ IS MORE; $\sin\theta = \frac{0.30}{0.33} \Rightarrow \theta = 65.4°$

SO, $\cos\theta = \frac{T_y}{T} \Rightarrow T = \frac{T_y}{\cos\theta} = \frac{7.5}{\cos 65.4} = \boxed{18.\ N = T}$

7. A car engine with weight 1000.0 N hangs from ropes as shown (the car engine is equilibrium so remember the forces must balance.) Find T_1 and T_2, the tensions in the two ropes shown.





$\Sigma\vec{F}_x = \vec{T}_2 + \vec{T}_{1x} = 0$

$\Sigma\vec{F}_y = \vec{T}_{1y} + \vec{w} = 0$

(SINCE IN EQUILIBRIUM).

$\sin 60.0° = \frac{T_{1y}}{T_1} \Rightarrow T_1 = \frac{T_{1y}}{\sin 60.0°} = \frac{1000.0}{0.866} = 1154.73\ N.$

$\tan 60.0° = \frac{T_{1y}}{T_{1x}} \Rightarrow T_{1x} = \frac{T_{1y}}{\tan 60.0°} = \frac{1000.0}{1.7321} = 577.4\ N$

$\therefore$ $T_1 = 1150\ N$

$T_2 = 577\ N$

$\vec{W}$ CAR + DRIVER $= 1.2\times10^4$ N $\Rightarrow$ $m = 1223$ kg.

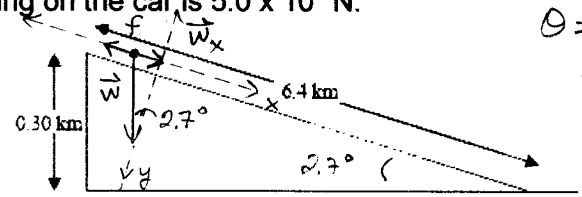
8. From the top of an incline, a road goes downwards in a straight line. At the top of the hill, a driver in a car stops to look at the view. In starting his journey down the hill, the driver decides to save fuel. He switches off the engine and allows the car to move freely down the hill. The car descends a height of 0.30 km in a distance of 6.4 km before levelling out, as shown. The average resistive force acting on the car is 5.0×10^2 N.

$\sin 2.7 = \frac{W_x}{W}$

$W_x = W \sin 2.7.$

$\theta = \sin^{-1}\left(\frac{0.30}{6.4}\right)$

$= 2.7°$



a) What is the acceleration of the car down the incline?

$\Sigma F_x = W_x - f = ma$

$\Rightarrow a = \frac{W_x - f}{m} = \frac{W\sin(2.7) - 5.0\times10^2}{1223} = \boxed{0.05\ \text{ms}^{-1}}$

b) What is the speed of the car at the bottom of the incline?

$v^2 = u^2 + 2as \Rightarrow v = (2as)^{1/2}$

$= (2(0.05)(6.4\times10^3))^{1/2} = 25.30\ \text{ms}^{-1} = \boxed{25\ \text{ms}^{-1}}$

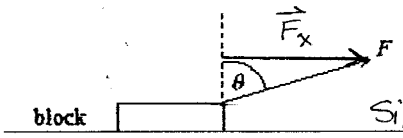
c) For the last few hundred meters of its journey down the hill, the car travels at constant speed. Determine the frictional force acting on the car whilst it is moving at constant speed.

AT THIS POINT, $f = W_x = W\sin(2.7)$

$= (1.2\times10^4)(\sin 2.7)$

$= 565.3$ N $= \boxed{560\text{N}}$

9. A block of mass m is pulled along a horizontal, frictionless surface by a force of magnitude F. The force makes an angle θ with the vertical. Determine an expression for the magnitude of the acceleration of the block in the horizontal direction produced by the force F, in terms of the given variables.



$F_x = ma_x$

$a_x = \frac{F_x}{m} = \boxed{\frac{F\sin\theta}{m}}$

$\sin\theta = \frac{F_x}{F}$

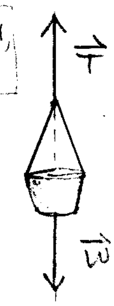
10. A rope with a bucket attached to the end is used to raise water from a well. The mass of the empty bucket is 1.2 kg and it can raise 10.0 kg of water when full. Find the tension in the rope when

a) the empty bucket is lowered with an acceleration of 2.0 ms^{-2}.

IN EQUILIBRIUM, T = W.

The apparent weight of the bucket is less, so $\vec{T}$ will be less;

$\Sigma F_y = +T - w = -ma \Rightarrow T = -ma + w = -(1.2)(2.0) + 11.8 = \boxed{9.4\text{ N}}$



b) the full bucket is raised with an acceleration of 0.30 ms^{-2}.

The apparent weight of the bucket is more, so $\vec{T}$ will be more;

$\Sigma F_y = +T - W = +ma \Rightarrow T = +ma + W = (11.2)(0.30) + 109.9$

$= 113.3\text{ N} = \boxed{110\text{ N}}$

EMPTY: $\vec{W} = 11.8\text{ N}$
FULL: $\vec{W} = 109.9\text{ N}$

11. A 4.80 kg bucket of water is accelerated upward by a cord of negligible mass whose breaking strength is 60.0 N. Find the maximum upward acceleration that can be given to the bucket without breaking the cord.

$\Sigma F_y = +T - W = +ma$ (SET T = 60N here)

$\Rightarrow a = \frac{T - W}{m} = \frac{60.0 - 47.1}{4.80} = 2.6875\ ms^{-2}$

$= \boxed{2.69\ ms^{-2}}$

$\vec{W} = (4.80)(9.81) = 47.1\text{ N}$

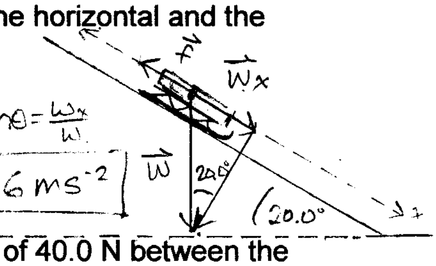
12. A sledge loaded with students (sledge and students have a total mass 150.0 kg) slides down a long, snow covered slope. The hill is at a constant angle of 20.0° above the horizontal and the sledge is well waxed so that there is no friction.

a) What is the sledge's acceleration?

$F_x = ma_x \Rightarrow a_x = \frac{F_x}{m} = \frac{W_x}{m} = \frac{W\sin\theta}{m} = \frac{mg\sin\theta}{m}$

$= (9.81)(\sin 20.0) = \boxed{3.36\ ms^{-2}}$

$\sin\theta = \frac{W_x}{W}$



b) Some sand is sprinkled on the slope, and it produces a frictional force of 40.0 N between the sledge and the slope. What is the sledge's acceleration?

$F_{NET} = W_x - f = ma \Rightarrow a = \frac{W_x - f}{m} = \frac{mg\sin\theta - f}{m}$

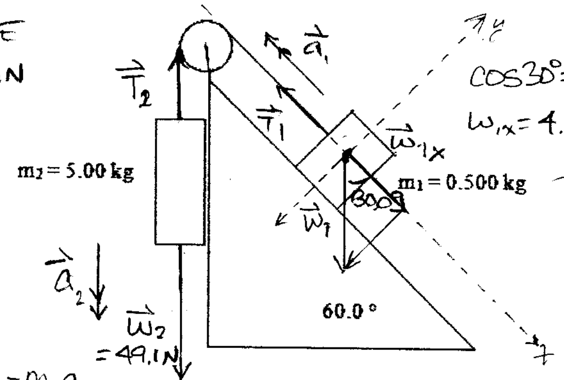
$= \frac{(150.0)(9.8)\sin(20.0) - 40.0}{150.0}$

$\boxed{a = 3.09\ ms^{-2}}$

13. Calculate the acceleration of the masses and the tension in the string in the diagram below. You can assume the slope is frictionless and the strings massless.

e

THE ACCELERATIONS OF BOTH MASSES ARE THE SAME, AND THE STRING HAS THE SAME TENSION THROUGHOUT. CLEARLY M_2 PULLS M_1 UP THE SLOPE, AND M_2 ACCELERATES DOWNWARD.



$\cos 30° = \frac{w_{1x}}{w_1}$

$w_{1x} = 4.25N$

SO FOR M_2, $\Sigma F_2 = +T_2 - w_2 = -m_2 a_2$ (1)

FOR M_1, $\Sigma F_1 = -T_1 + w_{1x} = -m_1 a_1$ (2)

2 EQNS, 2 UNKNOWNS: $a_1 = a_2$ and $T_1 = T_2$

$T = -m_2 a + w_2$ $\quad$ (1) → (2): $-(-m_2 a + w_2) + w_{1x} = -m_1 a$

$+m_2 a - w_2 + w_{1x} = -m_1 a$

(You can check your answers by using same eqns another way... ☺)

$a = \frac{w_2 - w_{1x}}{m_1 + m_2} = \frac{49.1 - 4.25}{5.5} = \boxed{8.2\ ms^{-2}}$

$T = -m_2 a + W_2$

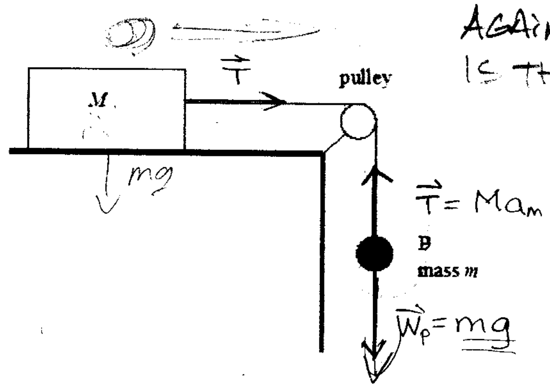
$= (5.00)(8.9) + 49.1$

$= \boxed{8.1N}$

14. A block on a frictionless horizontal table is attached by a light, inextensible string to an object P of mass m that hangs vertically as shown. The pulley has zero friction and the acceleration of free fall is g. Determine the acceleration of the block and object P in terms of the given variables.

c

AGAIN, ACCELERATIONS OF BLOCK AND OBJECT IS THE SAME (ASSUME STRING DOES NOT STRETCH).



FIND a OF MASS P:

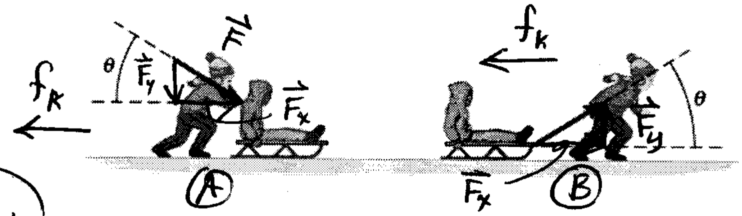
$\Sigma F_y = Ma_m - mg = -ma_P$

Since $a_M = a_P$,

$a(M+m) = mg$

$\Rightarrow \boxed{a = \frac{mg}{(m+M)}}$

15. A person has a choice of either pushing or pulling a sled at a constant velocity, as the drawing illustrates. Friction is present. If the angle is the same in both cases, does it require less force to push or to pull the sled? Explain. **[to pull]**



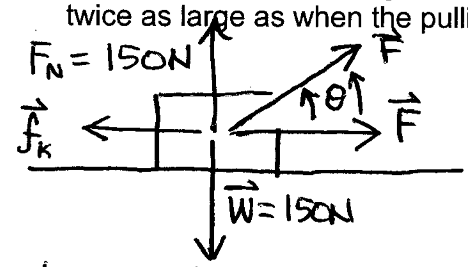
$\vec{f}_k = \mu_K \vec{F}_N$

TO PULL

IN CASE (A): The downward (at an angle) force when pushing adds to the sled's weight, thereby increasing F_N.
Since $f_k = \mu_k F_N$, the kinetic friction force INCREASES.

IN CASE (B): The upwards (at an angle) force when pulling reduces the sled's apparent weight, thereby decreasing F_N and f_k.

16. A box has a weight of 150 N and is being pulled across a horizontal floor by a force that has a magnitude of 110 N. The pulling force can point horizontally, or it can point above the horizontal at an angle θ. When the pulling force points horizontally, the kinetic frictional force acting on the box is twice as large as when the pulling force points at the angle θ. Find θ. **[43°]**



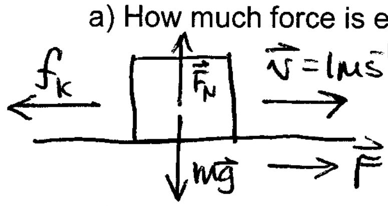
Horizontally: $f_k = \mu_k F_N = 150\mu_k$

When at an angle θ: $f_k' = \mu_k(150 - 110\sin\theta) = \frac{1}{2} f_k = 75\mu_k$

at an angle:
$F_N = 150 - 110\sin\theta$

Solve for θ:
$150 - 110\sin\theta = 75$
$\sin\theta = 0.68 \Rightarrow \boxed{\theta = 43°}$

17. A 25.0 kg wooden box is pushed across a wooden floor at a constant speed of 1.0 ms^{-1}.
a) How much force is exerted on the box? **[49 N]**



Since $a = 0$, $\Sigma F = 0$ } $f_k = F = \mu_k F_N$
$= \mu_k mg$
$= (0.3)(25.0)(9.81) = \boxed{74N}$

↑ depends on μ_k used...

b) If the force exerted on the box is doubled, what is the resulting acceleration of the box? **[2.0 ms^{-2}]**

Since $\vec{F} = m\vec{a}$, if $\vec{F}$ doubles then $\vec{a}$ doubles.
$2(1.0ms^{-2}) = \boxed{2.0ms^{-2}}$

c) How long would it take for the velocity of the crate to double to 2.0 ms^{-1}? **[0.50 s]**

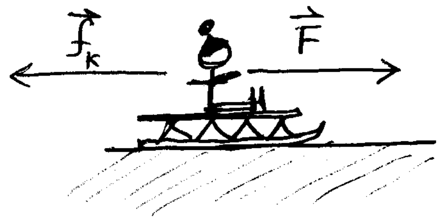
s	u	v	a	t
	+1.0	+2.0	+2.0	?

$v = u + at$

$t = \frac{v-u}{a} = \frac{2.0-1.0}{2.0} = \boxed{0.5s}$

... or simple reasoning could lead you to this. ☺

18. A boy exerts 36 N of force horizontally as he pulls a 5.3 kg sled across a cement sidewalk at constant speed. What is the coefficient of kinetic friction between the sled runner and the sidewalk (steel and cement)? **[0.69]**



Since $\Delta v = 0$, $a = 0$ and $\Sigma F = 0$

therefore, $f_k = F \Rightarrow \mu_k F_N = F$

$$\mu_k = \frac{F}{F_N} = \frac{F}{mg} = \frac{36}{(5.3)(9.81)} = \boxed{0.69}$$

19. Consider the previous problem, but with the sled moving over packed snow. The coefficient of friction (kinetic) is now only 0.12. If a person of mass 66.3 kg sits on the sled, what force is needed to pull the sled across the snow at constant speed? **[84 N]**

Again, $\Delta v = 0$ so $a = 0$ and $\Sigma F = 0$.

So, $f_k = F = \mu_k F_N = \mu_k mg = (0.12)(66.3+5.3)(9.81)$
$= \boxed{84\,N}$

20. A sled and rider of combined mass 50.0 kg is pulled along a flat, snow-covered ground. The static coefficient of friction between the steel runner and the snow is 0.30, and the kinetic friction coefficient is 0.10.

a) How much does the sled weigh? **[491 N]**

$W = mg = (50.0)(9.81) = \boxed{491\,N}$

b) What force will be needed to start the sled moving? **[147 N]**

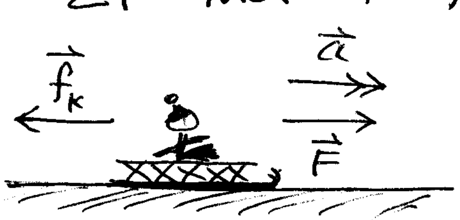
$f_{s(MAX)} = \mu_s F_N = (0.30)(491) = \boxed{147\,N}$

c) What force is necessary to keep the sled moving at constant velocity? **[49.1 N]**

$f_k = \mu_k F_N = (0.10)(491) = \boxed{49.1\,N}$

d) Once moving, what total force must be applied to the sled to accelerate it at 3.00 ms^{-2}? **[199 N]**

$\Sigma F = ma = F - f_k \Rightarrow F = ma + f_k$



$= ma + \mu_k mg$
$= m(a + \mu_k g)$
$= 50.0(3.00 + (0.10)(9.81)) = \boxed{199\,N}$