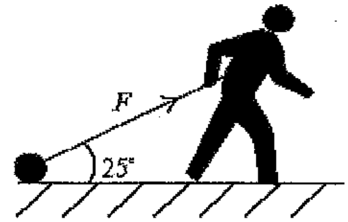
1. An athlete trains by dragging a heavy load across a rough horizontal surface. The athlete exerts a force of magnitude F on the load at an angle of 25° to the horizontal. Once the load is moving at a steady speed, the average horizontal frictional force acting on the load is 470 N. The load is moved a horizontal distance of 2.5 km in 1.2 hours. Calculate the work done on the load by the force F.

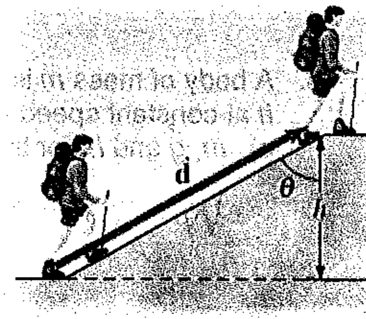


$W = Fs = (470)(2.5\times10^3) = 1175\times10^3 = \boxed{1.2\times10^6\ J}$

Note: the irrelevance of $\theta = 25°$ and $t = 1.2$ hours!

2. A hiker walks up a 150 m hill and then back down again.

a) Determine the amount of work the hiker must do on a 15.0 kg backpack while walking up to the summit of a 150 m hill.



h = 150 m. This is the VERTICAL DISTANCE to carry the backpack.

$W = Fs = ws = (mg)s = (15.0)(9.81)(150)$
$= \boxed{2.2\times10^4\ J}$

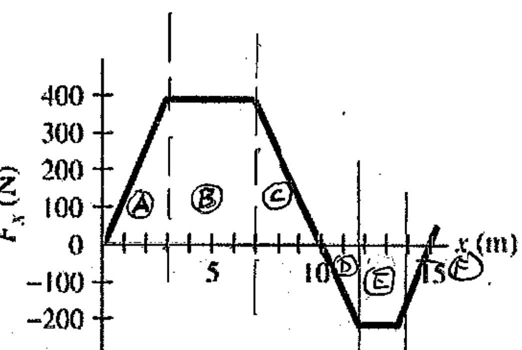
b) Determine the amount of work done on the backpack when walking from the summit back down to where the hiker started.

GRAVITY DOES THE WORK HERE.

$W = Fs = ws = (mg)s = \boxed{-2.2\times10^4\ J}$

NOTE: Total work done UP and DOWN = 0 J since $\vec{s} = 0$.

3. The force on an object, acting along the x axis, varies as shown. Determine the work done by this force to move the object



a) From 0.0 m to 10.0 m

A + B + C $= \frac{1}{2}(3\times400) + (4\times400) + \frac{1}{2}(3\times400)$
$= 600 + 1600 + 600$
$= \boxed{2800\ J}$

b) From 0.0 m to 15.0 m

SAME AS (a) but MINUS AREA (D+E+F).

(Work done = area between graph & x-axis)...

− D + E + F $= \frac{1}{2}(2\times200) + (2\times200) + \frac{1}{2}(1\times200)$

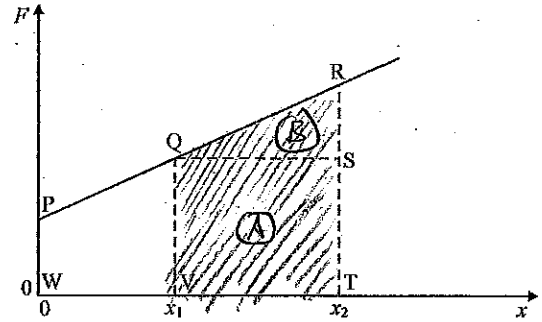
$= 200 + 400 + 100$

$= 700$ SO TOTAL WORK = 2800 − 700

4. The diagram below shows the variation with displacement x of the force F acting on an object in the direction of the displacement.

c

WORK DONE = AREA "UNDER" GRAPH.



Determine an expression for the work done by the force when the displacement changes from x_1 to x_2, in terms of the letter variables on the graph.

AREA IS : A + B = $\boxed{(VT \cdot ST) + \frac{1}{2}(QS \cdot RS)}$

5. A body of mass m is in a gravitational field of strength g. The body is moved through a distance h at constant speed v in the opposite direction to the field. Derive an expression in terms of
   a) m, g and h, for the work done on the body.

W = FORCE × DISTANCE = weight × distance

$= \boxed{mgh}$

   b) m, g and v, for the power required to move the body.

POWER = $\frac{\text{WORK}}{\text{TIME}} = mg\left(\frac{\Delta h}{\Delta t}\right) = \boxed{mgv}$

   c) A mass falls near the Earth's surface at constant speed in still air. Discuss the energy changes, if any, that occur in the gravitational potential energy and in the kinetic energy of the mass.

AS THE MASS FALLS, ITS GPE IS CONVERTED INTO KE.
TAKING h=0 AT GROUND LEVEL, WHEN IT HITS THE GROUND, ALL GPE HAS BEEN CONVERTED TO KE.
(IN REALITY, SOME ENERGY IS LOST TO AIR RESISTANCE + OTHER FACTORS).

6. An object of mass m_1 has a kinetic energy K_1. Another object of mass m_2 has a kinetic energy K_2. If the momentum of both objects is the same, determine the ratio $\frac{K_1}{K_2}$ in terms of the given variables.

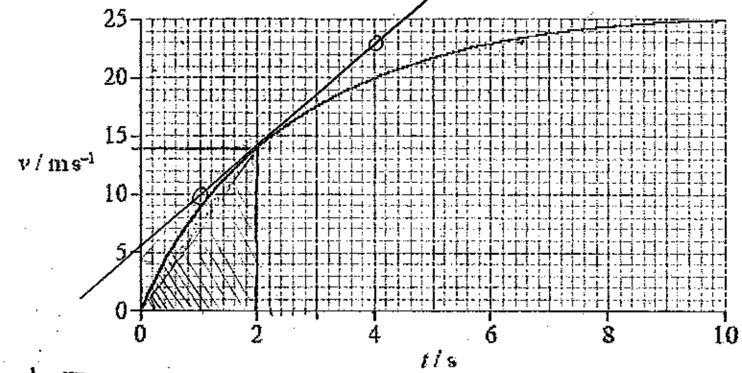
c

(*) $m_1 v_1 = m_2 v_2$ (SAME MOMENTUM) $\Rightarrow$ $K_1 = \frac{1}{2} m_1 v_1^2$

$K_2 = \frac{1}{2} m_2 v_2^2$

So, $\frac{K_1}{K_2} = \frac{\cancel{\frac{1}{2}}(\cancel{m_1 v_1})(v_1)}{\cancel{\frac{1}{2}}(\cancel{m_2 v_2})(v_2)} = \frac{v_1}{v_2}$, FROM (*), $\frac{v_1}{v_2} = \boxed{\frac{m_2}{m_1}}$

EACH BOX = (1)(0.2) = 0.2 m



7. The graph shows the variation with time t of the speed v of a ball of mass 0.50 kg that has been released from rest above the Earth's surface. The force of air resistance is **not** negligible.

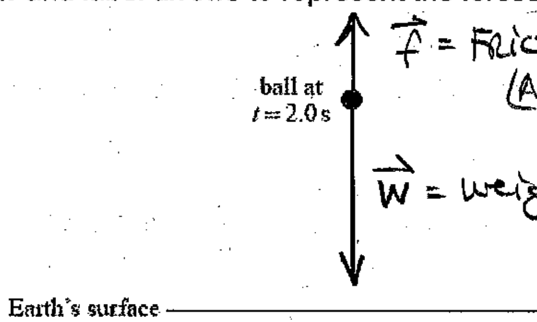
a) Determine the distance fallen after 2.0 s and the acceleration of the ball at 2.0 s.

DISTANCE = AREA UNDER CURVE
$\approx \boxed{15\,m}$

ACCELERATION = SLOPE OF TANGENT LINE
$\approx \frac{23-10}{4-1} \approx \boxed{4.3\ ms^{-2}}$

$a = \frac{v}{t}$, $v = \frac{d}{t} \Rightarrow d = vt.$

b) In the space below, draw and label arrows to represent the forces on the ball at 2.0 s.



$\vec{f}$ = FRICTIONAL FORCE (drag).
(AIR RESISTANCE)

$\vec{W}$ = weight

c) Calculate the magnitude of the force of air resistance on the ball at 2.0 s.

NET FORCE : $\Sigma F = -ma = f - mg$

$\Rightarrow f = -ma + mg = (0.5)(-4.3 + 9.81) = \boxed{2.8\ N}$

d) State and explain whether the air resistance on the ball at t = 5.0 s is smaller than, equal to **or** greater than the air resistance at t = 2.0 s.

Since $f = mg - ma$ and a decreases with time, at $t = 5.0\,s$ the air resistance is GREATER than it was at $t = 2.0\,s$.

e) After 10 s the ball has fallen 190 m. Show that the sum of the potential and kinetic energies of the ball has decreased by 780 J. Where has this energy gone?

$\Delta GPE = mg\Delta h = (0.50)(9.81)(190) = 932\,J$ = LOSS IN GPE

$\Delta KE = \frac{1}{2}m\Delta v^2 = \frac{1}{2}(0.50)(25)^2 = 156\,J$ = GAIN IN KE

LOSS IN GPE $\neq$ GAIN IN KE

DIFFERENCE IS $932 - 156 = 776\,J \approx 780\,J$ ✓

This energy has been lost through frictional forces between the air and the ball and dissipated mainly as heat (thermal energy transfer to both the ball and the surrounding atmosphere).

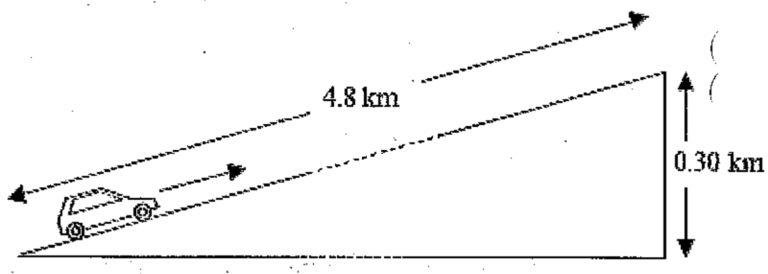
8. A car is travelling with constant speed v along a horizontal straight road. There is a total resistive force F acting on the car.

C

a) Deduce that the power P to overcome the force F is $P = Fv$.

$$\text{POWER} = \frac{\text{WORK}}{\text{TIME}} = \frac{F \cdot s}{t} = F\left(\frac{s}{t}\right) = \underline{Fv} \checkmark$$

b) The car drives up a straight incline that is 4.8 km long. The total height of the incline is 0.30 km. The car moves up the incline at a steady speed of 16 m s^{-1}. During the climb, the average friction force acting on the car is 5.0×10^2 N. The total weight of the car and the driver is 1.2×10^4 N.



c) Determine the time it takes the car to travel from the bottom to the top of the incline.

$$v = \frac{s}{t} \Rightarrow t = \frac{s}{v} = \frac{4.8\times10^3}{16} = \boxed{300\,s}$$

d) Determine the work done against the gravitational force in travelling from the bottom to the top of the incline.

$$W = \vec{F}\cdot\vec{s} = (mg)(s) = (1.2\times10^4)(0.30\times10^3) = \boxed{3.6\times10^6\,J}$$

e) Using your answers to (c) and (d), calculate a value for the minimum power output of the car engine needed to move the car from the bottom to the top of the incline.

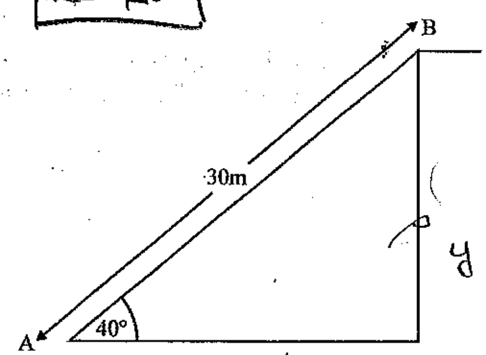
Work done by car to go up .30 km $= (mg)(s) = \underline{3.6\times10^6\,J}$.

work done by car to over come friction $= F\cdot s = (5.0\times10^2)(4.8\times10^3) = \underline{2.4\times10^6\,J}$.

TOTAL WORK $= 6.0\times10^6\,J \Rightarrow P = \frac{W}{t} = \frac{6.0\times10^6}{300} = \boxed{20\,kW}$

9. The diagram represents an escalator. People step on to it at point A and step off at point B. The escalator is 30 m long and makes an angle of 40° with the horizontal. At full capacity, 48 people step on at point A and step off at point B every minute.

a) Calculate the potential energy gained by a person of weight 7.0×10^2 N in moving from A to B.



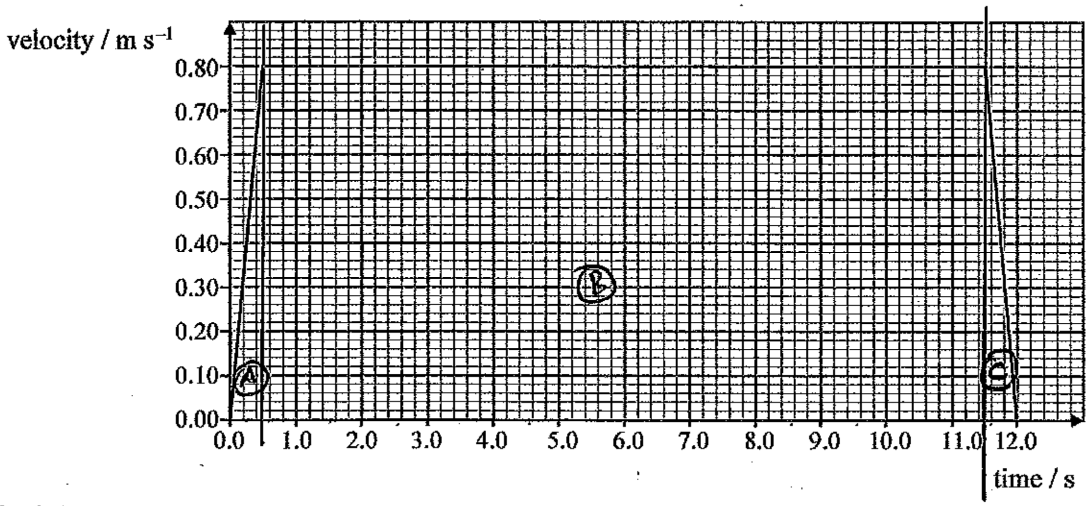
$$\Delta GPE = mg\Delta h = (7.0\times10^2)(30)(\sin 40°)$$
$$= \boxed{1.3\times10^4\,J}$$

$$\sin 40° = \frac{y}{30}$$

b) Estimate the energy supplied by the escalator motor to the people every minute when the escalator is working at full capacity.

$$\Delta GPE \text{ AT FULL CAPACITY} = (1.3\times10^4)(48) = 6.24\times10^5\,J$$
$$= \boxed{6.2\times10^5\,J}$$

10. An elevator (lift) starts from rest on the ground floor and comes to rest at a higher floor. Its motion is controlled by an electric motor. A simplified graph of the variation of the elevator's velocity with time is shown below. The mass of the elevator is 250 kg.



Calculate:

a) the acceleration of the elevator during the first 0.50 s.

$a = \frac{\Delta v}{\Delta t}$ = gradient of v-t graph = $\frac{0.80 - 0.00}{0.50}$ = $\boxed{1.6 \text{ ms}^{-2}}$

b) the total distance travelled by the elevator.

AREA UNDER ENTIRE GRAPH = A + B + C

$= \frac{1}{2}(0.80)(0.5) + (0.80)(11.0) + \frac{1}{2}(0.80)(0.50)$

$= 0.2 + 8.8 + 0.2 = \boxed{9.2 \text{ m}}$

c) the minimum work required to raise the elevator to the higher floor.

$W = F \cdot s = (mg)(\Delta h) = (250)(9.81)(9.2) = 22{,}563 \text{ J}$

$= \boxed{23 \text{ kJ}}$

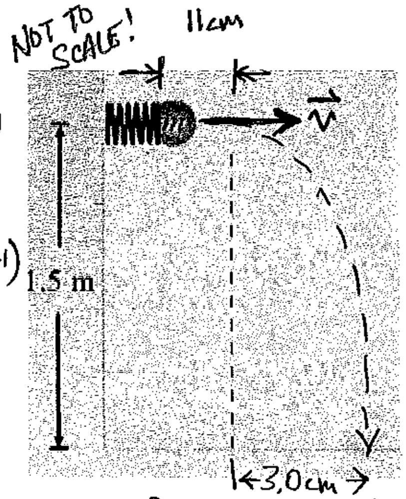
d) the minimum average power required to raise the elevator to the higher floor.

POWER = $\frac{\text{WORK}}{\text{TIME}} = \frac{23 \times 10^3}{12.0} = 1880 \text{ W} = \boxed{1.9 \text{ kW}}$

e) the efficiency of the electric motor that lifts the elevator, given that the input power to the motor is 5.0 kW.

EFFECIENCY = $\frac{\text{OUTPUT POWER}}{\text{INPUT POWER}} = \frac{1.9}{5.0} = \boxed{.38}$

11. A spring is attached horizontally to a wall as shown. 1.5 m above the floor. A small ball of mass 50.0 g is pushed onto the spring, compressing it, then released. If the horizontal displacement of the ball is 3.0 cm and the spring is compressed 11 cm before released, determine the spring constant k of the spring. **[~~121 Nm^-1~~]** $(1.21\times10^{-2}\,Nm^{-1})$



TIME TO HIT FLOOR:

s	u	v	a	t
1.5	0		9.81	

$S = \frac{1}{2}at^2 \quad t = \sqrt{\frac{2s}{a}} = \underline{0.553s}$

SPEED WHEN BALL LEAVES SPRING:

$v_x = \text{constant} = \frac{\Delta s}{\Delta t} = \frac{0.03}{0.553} = 0.05425\,ms^{-1} = \Delta v.$

$\Delta E_p = \Delta KE \Rightarrow \frac{1}{2}k\Delta x^2 = \frac{1}{2}m\Delta v^2 \Rightarrow k = \frac{m\Delta v^2}{\Delta x^2} = \frac{(0.0500)(0.05425)^2}{(0.11)^2} = \boxed{0.0121\,Nm^{-2}}$

(ELASTIC PE) = (KE OF BALL)

12. From the top of a cliff 80.0 meters high, a football of mass 0.700 kilogram is dropped vertically from rest.

a) Calculate the potential, kinetic, and mechanical (total) energies of the ball at time t = 0 s. **[549 J, 0 J, 549 J]**

TAKE BOTTOM OF CLIFF AS h = 0m.

then: $GPE = mgh = (0.700)(9.81)(80.0) = \boxed{549J}$

$KE = 0J$ SINCE AT REST

$ME = GPE + KE = 549J + 0J = \boxed{549J}$

b) Calculate the potential, kinetic, and mechanical (total) energies of the ball at the *instant just before it hits the ground.* **[0J, 549 J, 549 J]**

$GPE = 0J$

$KE = 549J$ (by conservation of energy)

$ME = GPE + KE = 0J + 549J = \boxed{549J}$

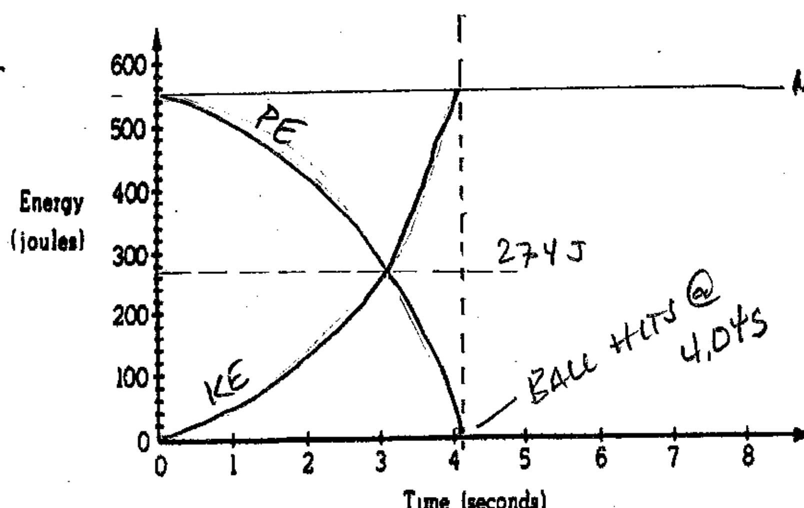
c) On the axes below, draw and label the kinetic, potential, and mechanical (total) energies of the ball as functions of time until the ball hits.

s	u	v	a	t
80.0	0		-9.81	?

$S = \frac{1}{2}at^2$

$t = \sqrt{\frac{2s}{a}}$

$= \sqrt{\frac{2(80.0)}{9.81}} = \underline{4.04s}$



ME = CONSTANT = 549J

NOTE: at h=40m, KE = PE

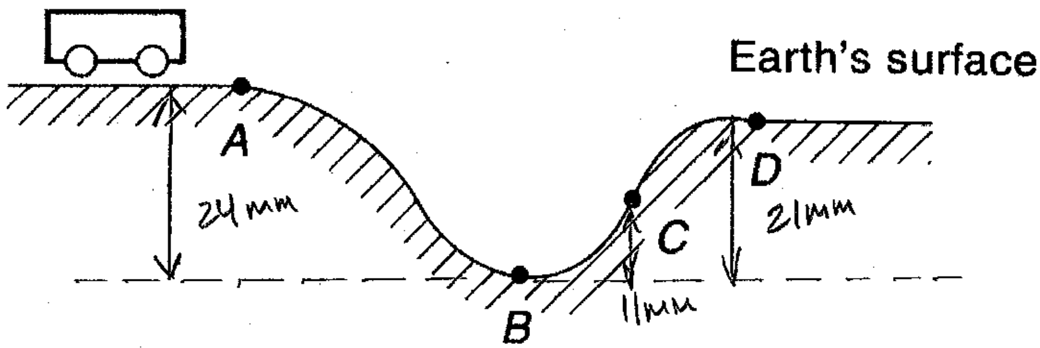
HEIGHT (m)	PE(J)	KE(J)	TIME(s)
70.0	480	69	1.4
60.0	412	137	2.0
50.0	343	206	2.5
40.0	274	275	2.9
30.0	206	343	3.2

SPREADSHEET!

d) Explain in detail how the answers to questions 1-3 would differ (if at all) if the ball were kicked horizontally at a certain speed instead of dropped straight down from rest.

The GPE would not change, and would vary with time exactly as shown in the graph, since vertical and horizontal components of freefall are independent.
HOWEVER: KE would be greater at every point, (since there would now be and additional horizontal component to the ball's velocity).
But the shape of the KE graph would remain the same.

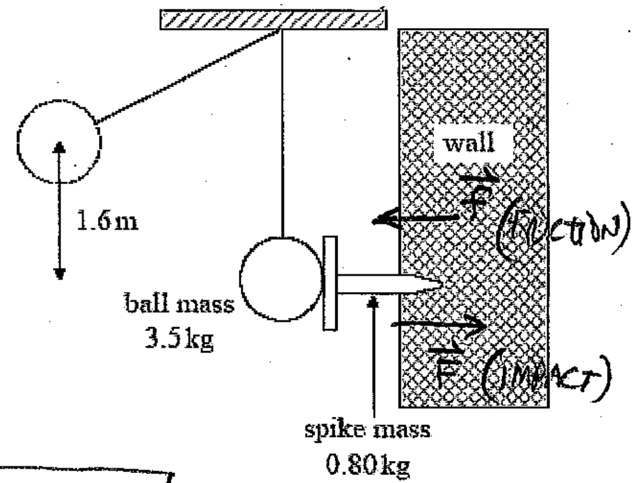
13. A 2.0 kg cart is placed on a frictionless track at point A in the diagram, and released from rest. Assume the gravitational potential energy of the mass to be zero at point B. The scale of the diagram is 1 mm = 1 m. Fill in the table below.



POINT	v (m/s)	h (m)	KE (J)	PE (J)	ME (J)
A	0	24	0	471	471
B	22	0	471	0	471
C	16	11	255	216	471
D	7.7	21	59	412	471

14. A large swinging ball is used to drive a horizontal iron spike into a vertical wall. The centre of the ball falls through a vertical height of 1.6 m before striking the spike in the position shown.

The mass of the ball is 3.5 kg and the mass of the spike is 0.80 kg. Immediately after striking the spike, the ball and spike move together.



a) Determine the speed of the ball when it strikes the spike. **[5.6 ms^{-1}]**

$\Delta KE = \Delta GPE$

$\frac{1}{2}\cancel{m}v^2 = \cancel{m}gh \Rightarrow v = \sqrt{2gh} = \sqrt{2(9.81)(1.6)}$

$= \boxed{5.6 ms^{-1}}$

$v_{(ball+spike)} = 4.6 ms^{-1}$

b) Determine the energy dissipated as a result of the collision. **~~[10 J]~~** 9.4

KE just before collision $= \frac{1}{2}mv^2 = \frac{1}{2}(3.5)(5.6)^2 = 54.88 J$

KE just after collision $= \frac{1}{2}m'v'^2 = \frac{1}{2}(3.5+0.80)(4.6)^2 = 45.49 J$

The difference is: $54.88 - 45.49 = \boxed{9.4 J}$

c) As a result of the ball striking the spike, the spike is driven a distance 7.3×10^{-2} m into the wall. Calculate, assuming it to be constant, the friction force F between the spike and wall.

[6.2 ×10^2 N]

$W = \Delta KE = Fs$

$F = \frac{\Delta KE}{s} = \frac{45.49}{7.3 \times 10^{-2}} = \boxed{620 N}$

– or –

$v^2 = u^2 + 2as$

$= \cancel{u^2}^{0} + 2\frac{Fs}{m}$

$\Rightarrow F = \frac{mv^2}{2s} = \frac{(4.3)(4.6)^2}{2(7.3\times10^{-2})} = \boxed{620 N}$

d) The machine that is used to raise the ball has a useful power output of 18 W. Calculate how long it takes for the machine to raise the ball through a height of 1.6 m. **[3.1 s]**

$P = \frac{W}{t} = \frac{mgh}{t} \Rightarrow t = \frac{mgh}{P} = \frac{(3.5)(9.81)(1.6)}{18}$

$= \boxed{3.1 s}$