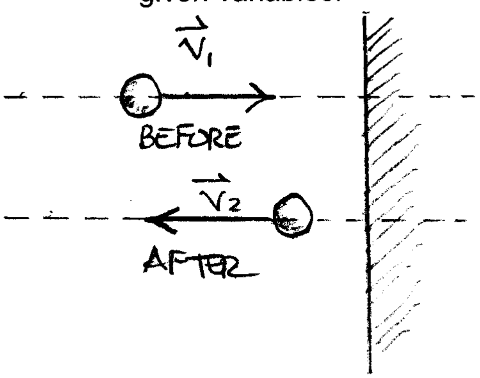
HOMEWORK PROBLEMS:

1. A ball of mass m, travelling in a direction at right angles to a vertical wall, strikes the wall with a speed v_1. It rebounds at right angles to the wall with a speed v_2. The ball is in contact with the wall for a time Δt. Write an expression for the magnitude of the force that the ball exerts on the wall in terms of these given variables. $[\frac{m(v_1+v_2)}{\Delta t}]$



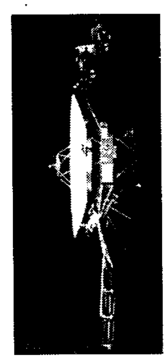
$\Delta v = \vec{v}_2 - \vec{v}_1$
$= \vec{v}_2 + -\vec{v}_1$

Since $\vec{v}_1$ and $\vec{v}_2$ are opposite directions, $\Delta v = v_2 + v_1$

then, $F\Delta t = m\Delta v$

$\Rightarrow F = \frac{m\Delta v}{\Delta t} = \boxed{\frac{m(v_2+v_1)}{\Delta t} = F}$

2. The Voyager 1 spacecraft was launched on September 5, 1977 from Cape Canaveral, Florida, USA. Its dry mass (with no fuel) was measured as 721.9 kg. During its journey through the solar system, it explored all of the outer giant planets, 48 of their moons, and various rings and magnetic fields. Voyager 1 crossed into the final frontier of the solar system, and into interstellar deep space, around August 1, 2002 at a speed of 21.725 km/s. Its fuel was exhausted decades ago but to this day it continues to fly through the vacuum of deep space. Scientists hope to maintain radio contact with the spacecraft until the year 2020.



a) Assuming that no external forces act upon this spaceship, predict how fast it will be going in 100 years. Explain your answer using Newton's Laws as appropriate. **[21.725 kms^{-1}]**

$\boxed{21.725\ km\,s^{-1}}$; the speed and velocity will NOT change. Newton's First Law states that "when no forces act on a body, that body will either remain at rest or continue to move along a straight line at constant speed."

b) Speculate on at least 2 possible (and credible) sources of external forces that could act on the spaceship, causing it to change its motion according to Newton's First Law.

(1) the gravitational force of a planet, star, comet, asteriod etc.
(2) the force of impact during a collision with another body (such as listed in (1)).

c) Determine the force necessary to bring Voyager 1 to a complete stop in exactly 1 hour. **[4357 N]**

$F\Delta t = m\Delta v \Rightarrow F = \frac{m\Delta v}{\Delta t}$

$= \frac{(721.9)(21.725\times10^3)}{3600}$

$= \boxed{4357\ N}$

m_c $\qquad$ v_c

3. A compact car, with a mass of 725 kg, is moving at +100.0 km hr^{-1}. At what velocity is the momentum of a larger car (mass 2175 kg) equal to that of the smaller car?

$m_c = 725\ kg$
$v_c = +100.0\ km\ hr^{-1}$
$m_l = 2175\ kg$
$v_l = ?$

Momentum is the same for each:

$$m_c v_c = m_l v_l$$

$$v_l = \frac{m_c v_c}{m_l} = \frac{(725)(+100.0)}{(2175)} = \boxed{33.3\ km\ hr^{-1}}$$

4. A snowmobile has a mass of 2.50 × 10^{2} kg. A constant force is exerted on it for 60.0 s. the snowmobile's initial velocity is 6.00 ms^{-1} and its final velocity is 28.0 ms^{-1}.

a) What is the snowmobile's change in momentum?

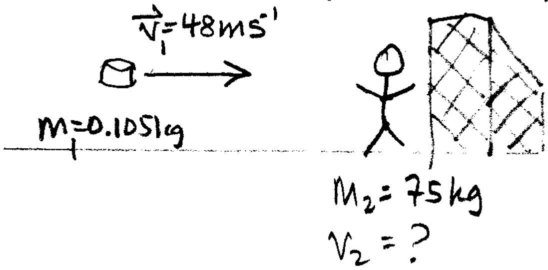
$$\Delta p = m\Delta v = m(v-u) = (2.50\times10^{2})(28.0-6.00) = \boxed{5500\ kg\,ms^{-1}}$$

b) What is the magnitude of the force exerted on it?

$$F\Delta t = m\Delta v = \Delta p \Rightarrow F = \frac{\Delta p}{\Delta t} = \frac{5500}{60.0} = \boxed{91.7\ N}$$

FROM THE ENGINE, OF COURSE!

5. A 0.105 kg hockey puck moving at 48 ms^{-1} is caught by a 75 kg goalie at rest on a frictionless ice surface. After impact, with what speed does the goalie slide backwards on the ice?



CONS. OF MOMENTUM:

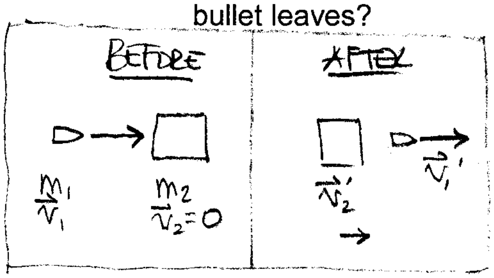
$$m_1 v_1 + \cancel{m_2 v_2}^{\,0} = m_1 v_1' + m_2 v_2' = (m_1+m_2)v'$$

After, $v_1' = v_2' = v'$

SO,

$$v' = \frac{m_1 v_1}{m_1+m_2} = \frac{(48)(0.105)}{75.105} = \boxed{0.67\ ms^{-1}}$$

6. A 35.0 g bullet moving at 475 ms^{-1} strikes a 2.5 kg wooden block. The bullet passes through the block, leaving at 275 ms^{-1}. The block was at rest when the bullet hit. How fast is it moving when the bullet leaves?



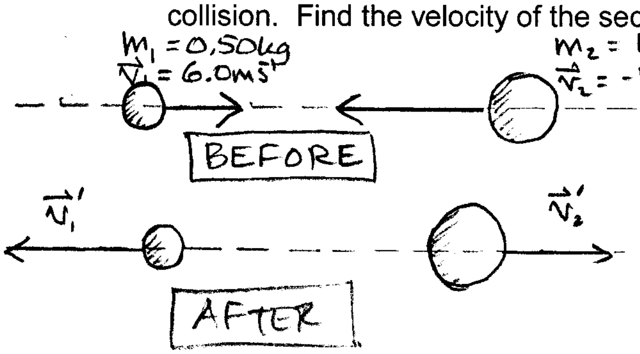
CONS. OF MOMENTUM

$$m_1 v_1 + \cancel{m_2 v_2}^{\,0} = m_1 v_1' + m_2 v_2'$$

$$v_2' = \frac{m_1 v_1 - m_1 v_1'}{m_2} = \frac{(0.0350)(475)-(0.350)(275)}{2.5}$$

$$= \boxed{2.8\ ms^{-1}}$$

7. A 0.50 kg ball traveling at +6.0 ms^{-1} collides head-on with a 1.00 kg ball moving in the opposite direction at a velocity of -12.0 m s^{-1}. The 0.50 kg ball moves away at -14.0 m s^{-1} after the collision. Find the velocity of the second ball after the collision.



CONS. OF MOMENTUM

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$v_2' = \frac{m_1 v_1 + m_2 v_2 - m_1 v_1'}{m_2}$$

$$= \frac{m_1(v_1 - v_1') + m_2 v_2}{m_2} = \frac{(0.50)(+6.0-^-14.0)+(1.00)(-12.0)}{1.00}$$

$$= \boxed{-2.0\ ms^{-1}}$$

MOVING TO THE LEFT!

IN ALL CASES, $m_1 = 1\,kg$ cart, $m_2 = 2\,kg$ cart

8. Two carts are rolling along a frictionless track. One cart has mass $m_1 = 1.0$ kg, the other $m_2 = 2.0$ kg $= 2m_1$. For each scenario, state the type of collision, and determine the unknown velocity **v**. For each collision, find out how much kinetic energy is lost. *Vectors drawn are not necessarily to scale.*

a)

[v = +3.0 ms^{-1}, 9.0 J lost]

$$m_1v_1 + \cancel{m_2v_2}^{0} = \cancel{m_1v_1'}^{0} + m_2v_2'$$

$$v_2' = \frac{m_1v_1}{m_2} = \frac{(1)(6.0)}{(2)} = \boxed{3.0\ ms^{-1}}$$

$$KE = \tfrac{1}{2}m_1v_1^2 = \tfrac{1}{2}(1.0)(6.0)^2 = 18\,J$$

$$KE' = \tfrac{1}{2}m_2v_2'^2 = \tfrac{1}{2}(2.0)(3.0)^2 = 9.0\,J$$

$$\therefore \Delta KE = KE' - KE = 18 - 9.0 = \boxed{9.0\,J}$$

ELASTIC, BUT NOT "PERFECTLY"

b)

[v = +2.0 ms^{-1}, 12 J lost]

$$m_1v_1 + \cancel{m_2v_2}^{0} = m_1v_1' + m_2v_2'$$

$$m_1v_1 = v'(m_1 + m_2)$$

$$v' = \frac{m_1v'}{m_1+m_2} = \frac{(1.0)(6.0)}{1.0 + 2.0} = \boxed{+2.0\,ms^{-1}}$$

$$KE = \tfrac{1}{2}m_1v_1^2 = \tfrac{1}{2}(1.0)(6.0)^2 = 18\,J$$

$$KE' = \tfrac{1}{2}(m_1+m_2)v'^2 = \tfrac{1}{2}(3.0)(2.0)^2 = 6.0\,J$$

$$\therefore \Delta KE = KE' - KE = \boxed{12\,J}$$

INELASTIC

c)

[v = 0 ms^{-1}, 27 J lost]

"PERFECTLY" INELASTIC

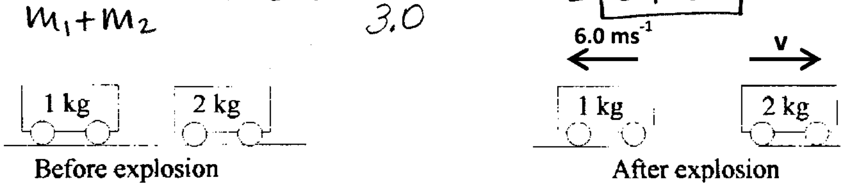
$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2' = (m_1 + m_2)v'$$

$$v' = \frac{m_1v_1 + m_2v_2}{m_1+m_2} = \frac{(1.0)(6.0) + 2.0(-3.0)}{3.0} = \boxed{0\,ms^{-1}}$$

$$KE = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 = \tfrac{1}{2}(1.0)(6.0)^2 + \tfrac{1}{2}(2.0)(-3.0)^2 = 27\,J$$

$$KE' = 0\,J \quad \therefore \boxed{\Delta KE = 27\,J}$$

d)



[v = +3.0 ms^{-1}, 0 J lost]

$$0 = m_1v_1' + m_2v_2'$$

$$v_2' = \frac{-m_1v_1'}{m_2} = \frac{-(1.0)(-6.0)}{2.0} = \boxed{+3.0\,ms^{-1}}$$

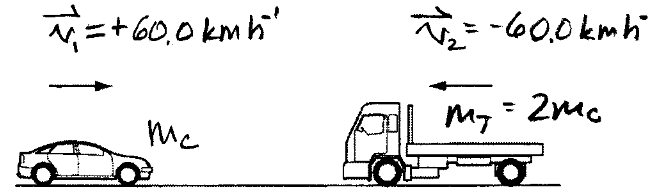
$KE = 0\,J$

NO KE LOST, SINCE NONE TO BEGIN WITH!

NOTE $KE' = \tfrac{1}{2}m_1v_1'^2 + \tfrac{1}{2}m_2v_2'^2 = \tfrac{1}{2}(1.0)(6.0)^2 + \tfrac{1}{2}(2.0)(3.0)^2 = \boxed{27\,J}$

THIS IS NOT A COLLISION. (27 J GAINED)

9. A car and a truck are both travelling at the speed limit of 60.0 kmh^{-1} but in opposite directions as shown. The truck has twice the mass of the car. The vehicles collide head-on and become entangled together.



a) During the collision, how does the force exerted by the car on the truck compare with the force exerted by the truck on the car? Explain.

By Newton's 3rd law, $F_c = F_T$. The forces are the same.

THIS MEANS THAT THE VELOCITY OF THE CAR WILL CHANGE MORE THAN THE VELOCITY OF THE TRUCK!

b) In what direction will the entangled vehicles move after the collision, or will they be stationary? Explain.

they will move to the left, the greater mass (the truck) is coming from the right, and therefore has greater inertia.

c) Determine the speed of the combined wreck immediately after the collision.

Assuming momentum is conserved and collision is inelastic:

$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2' = v'(m_1+m_2)$

$$v' = \frac{m_1v_1 + m_2v_2}{(m_1+m_2)} = \frac{m_1(60.0) + 2m_1(-60.0)}{3m_1} = \frac{-60.0\cancel{m_1}}{3\cancel{m_1}} = \boxed{-20.0\ \mathrm{ms^{-1}}}$$

d) How does the acceleration of the car compare with the acceleration of the truck during the collision? Explain.

Forces are the same, by Newton's 3rd law.

$F_c = m_c a_c,\ F_T = m_T a_T \Rightarrow m_c a_c = m_T a_T = 2m_c a_T$

$\therefore a_c = 2a_T$ $\boxed{\therefore \text{ CAR HAS TWICE THE ACCELERATION AS THE TRUCK}}$

e) Both the car and truck drivers are wearing seat belts. Which driver if more likely to be severely jolted in the collision? Explain.

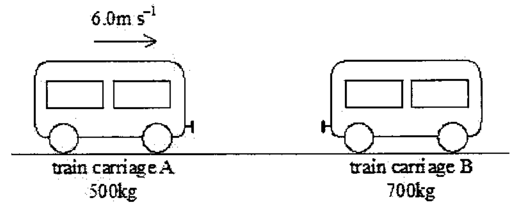
The car driver, because his $a = \frac{\Delta v}{\Delta t}$ is much greater.

(The car driver goes from $+\vec{v}$ to a $-\vec{v}$).

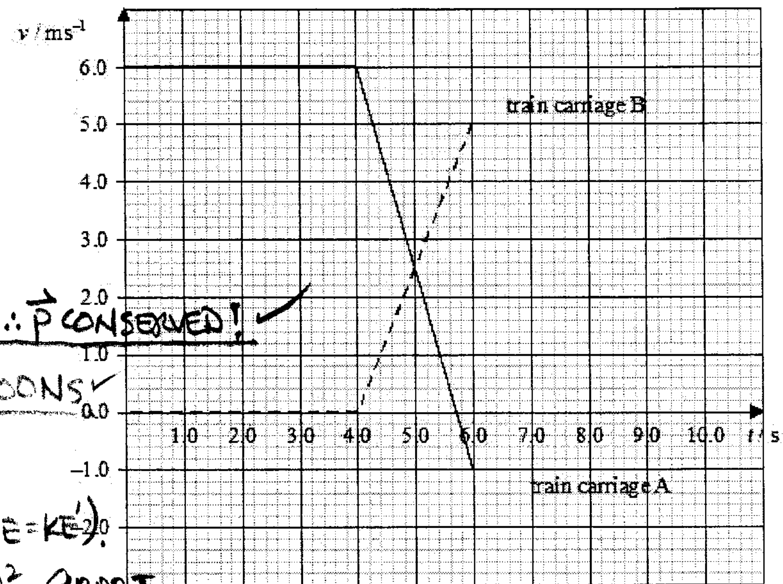
f) The total kinetic energy of the system decreases as a result of the collision. Is the principle of conservation of energy violated? Explain.

NO, because energy goes into deforming the cars as well as being dissipated as heat through friction.

10. A train carriage A of mass 500 kg is moving horizontally at 6.0 m s^{-1}. It collides with another train carriage B of mass 700 kg that is initially at rest, as shown in the diagram. The graph shows the variation with time t of the velocities of the two train carriages before, during and after the collision.



a) Use the graph to deduce that the total momentum of the system is conserved in the collision.



IF $\vec{p}$ is conserved, $p = p'$

$p = m_1 v_1 + \cancel{m_2 v_2}$ $= (500)(6.0) = \underline{3000\ \text{Ns}}$ ✓

$p' = m_1 v_1' + m_2 v_2'$ $= (500)(-1.0) + (700)(5.0) = \underline{3000\ \text{Ns}}$ ✓

$\therefore \vec{p}$ CONSERVED! ✓

b) Use the graph to deduce that the collision is elastic.

IF ELASTIC, NO ENERGY LOSS ($KE = KE'$)

$KE = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 \cancel{v_2^2} = \frac{1}{2}(500)(6.0)^2 = \underline{9000\ J}$

$KE' = \frac{1}{2}m_1 v_1'^2 + \frac{1}{2}m_2 v_2'^2 = \frac{1}{2}(500)(-1.0)^2 + \frac{1}{2}(700)(5.0)^2 = \underline{9000\ J}$

Since $KE = KE'$, the collision is elastic. ✓

c) Calculate the magnitude of the average force experienced by train carriage B. **[1800 N]**

$F\Delta t = m\Delta v$

$\Rightarrow F = \frac{m\Delta v}{\Delta t} = \frac{(700)(5.0)}{2.0} = 1750\ N = \boxed{1800 N}$

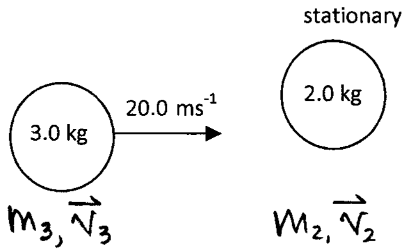
Note that GRADIENT $= \frac{\Delta v}{\Delta t} = \frac{F}{m} \Rightarrow \underline{F = 1750 N}$

11. Two balls collide as shown. Determine the angle α and the velocity **v** of the 3.0 kg ball.

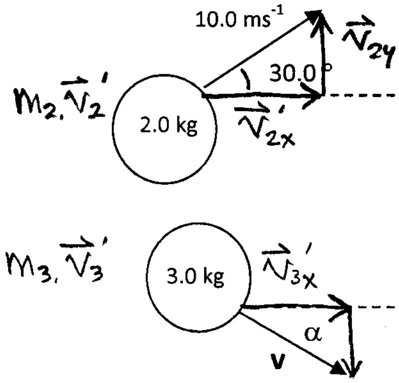
[v = 15 ms^{-1}, α = 13°]

Assume $\vec{p}$ is conserved.

$\vec{p}_x = \vec{p}_x{}'$ and $\vec{p}_y = \vec{p}_y{}'$



BEFORE COLLISION



AFTER COLLISION

$\sin 30.0° = \dfrac{v_{2y}{}'}{v_2{}'} \Rightarrow v_{2y}{}' = v_2{}' \sin 30.0°$

$v_{2x}{}' = v_2{}' \cos 30.0°$

$v_{3y}{}' = v_3{}' \sin\alpha$

$v_{3x}{}' = v_3{}' \cos\alpha$

X-DIR: $\vec{p} = m_3 \vec{v}_{3x}$
$= (3.0)(20.0)$
$= 60.0 \text{ Ns}$

Y-DIR: $\vec{p}_y = 0 \text{ Ns}$

X-DIR: $\vec{p}_x{}' = m_2 v_{2x}{}' + m_3 v_{3x}{}'$
$= (2.0)(10.0)(\cos 30.0°) + (3.0)(v_3{}')\cos\alpha$
$\vec{p}_x{}' = 17.321 + 3.0 v_3{}' \cos\alpha$

Y-DIR: $\vec{p}_y{}' = m_2 p_{2y}{}' + m_3 p_{3y}{}'$
$= (2.0)(10.0)(\sin 30.0°) + {}^{-}(3.0) v_3{}' \sin\alpha$
$\vec{p}_y{}' = 10.0 - 3.0 v_3{}' \sin\alpha$

NOW, $\vec{p}_x = \vec{p}_x$

(1) $60.0 = 17.321 + 3.0 v_3{}' \cos\alpha$

$\vec{p}_y = \vec{p}_y{}'$

(2) $0 = 10.0 - 3.0 v_3{}' \sin\alpha$

2 EQUATIONS, 2 UNKNOWNS...

FROM (1): $v_3{}' = \dfrac{60.0 - 17.321}{3.0 \cos\alpha}$

$v_3{}' = \dfrac{14.226}{\cos\alpha}$

INTO (2)

$0 = 10.0 - 3.0\left(\dfrac{14.226}{\cos\alpha}\right)\sin\alpha$

$0 = 10.0 - 42.678 \tan\alpha$

$\tan\alpha = 0.2343$

$\alpha = \tan^{-1}(0.2343)$

$= 13.187° = \boxed{13°}$

THEN, INTO (2):

$v_3{}' = \dfrac{10.0}{3.0 \sin\alpha} = \dfrac{10.0}{3.0 \sin 13°}$

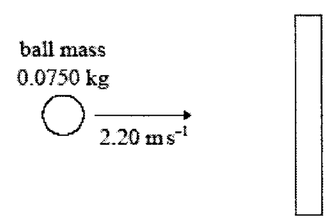
$= 14.61 \text{ ms}^{-1} = \boxed{15 \text{ ms}^{-1}}$

12. Fill in the following table, given typical masses and speeds of the following objects:

OBJECT	MASS (kg)	SPEED (ms^{-1})	MOMENTUM ($kgms^{-1}$)	FORCE TO STOP IN 60s (N)	ACCELERATION (ms^{-2})
electron	9.1×10^{-31}	3.0×10^{8}	2.7×10^{-22}	4.6×10^{-24}	5.0×10^{6}
oil tanker	5.5×10^{8}	1.5×10^{-2}	8.3×10^{6}	1.4×10^{5}	2.5×10^{-4}
rain drop	6.0×10^{-4}	1.0×10^{1}	6.0×10^{-3}	1.0×10^{-4}	1.7×10^{-1}
snail	4.7×10^{-2}	2.1×10^{-4}	9.9×10^{-6}	1.6×10^{-7}	3.5×10^{-6}
satellite	7.0×10^{0}	8.5×10^{4}	6.0×10^{5}	9.9×10^{3}	1.4×10^{3}
human runner	7.3×10^{1}	3.9×10^{0}	2.8×10^{2}	4.7	6.5×10^{-2}

$p = mv$ $\quad\quad F = \frac{m\Delta v}{\Delta t} = \frac{\Delta p}{\Delta t} = ma \Rightarrow a = \left(\frac{\Delta p}{\Delta t}\right)\frac{1}{m}$

13. A ball of mass 0.0750 kg is travelling horizontally with a speed of 2.20 m s^{-1}. It strikes a vertical wall and rebounds horizontally. Due to the collision with the wall, 20 % of the ball's initial kinetic energy is dissipated.



a) Determine the speed with which the ball rebounds from the wall. **[1.97 m s^{-1}]**

$KE = \frac{1}{2}mv^2 = \frac{1}{2}(0.0750)(2.20)^2 = 0.1815J$

20% OF THIS IS: $(.20)(0.1815) = 0.0363J$

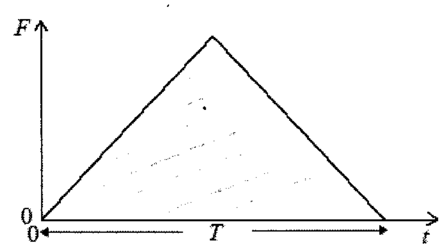
$KE' = 0.1815 - 0.0363 = 0.1452 = \frac{1}{2}mv'^2 \Rightarrow v' = \sqrt{\frac{2(0.1452)}{0.0750}} = \boxed{1.97ms^{-1}}$

b) Determine the impulse given to the ball by the wall. **[0.313 N s]**

$\vec{F}\Delta t = |m\Delta\vec{v}| = |m(\vec{v}_2 - \vec{v}_1)|$

$= |0.0750(-1.97 - 2.20)| = \boxed{0.313Ns}$

c) The sketch graph shows how the force *F* that the wall exerts on the ball is assumed to vary with time *t*. The time *T* is measured electronically to equal 0.0894 s. Determine the average value of *F*. **[3.50 N]**



$F\Delta t = m\Delta v$

$\Rightarrow F = \frac{m\Delta v}{\Delta t} = \frac{0.313}{0.0894} = \boxed{3.50N}$

** ALSO, AREA UNDER GRAPH = IMPULSE

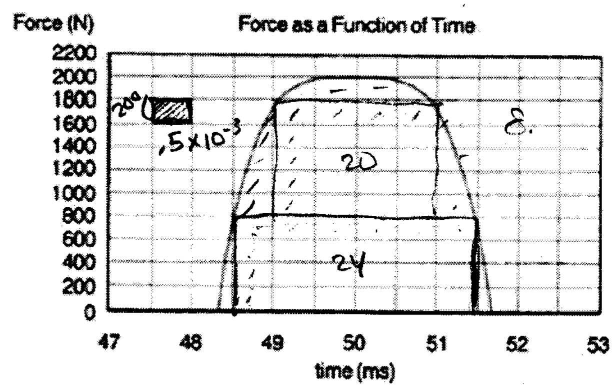
$\frac{1}{2}F_{MAX}T = 0.313$

$F_{MAX} = \frac{2(0.313)}{0.0894} = 7.00N$

$F_{AVE} = \frac{1}{2}F_{MAX} = \frac{1}{2}(7.00) = \boxed{3.50N}$

ONE SQUARE $= (200)(0.5 \times 10^{-3}) = 0.10$

14. The graph the force applied against time during a collision. The collision involved one ball hitting another ball initially at rest.



a) Find the impulse given to the ball at rest by the moving ball.

$F\Delta t$ = impulse = area under graph

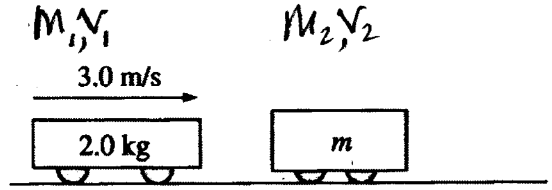
$\approx \boxed{52\ Ns}$

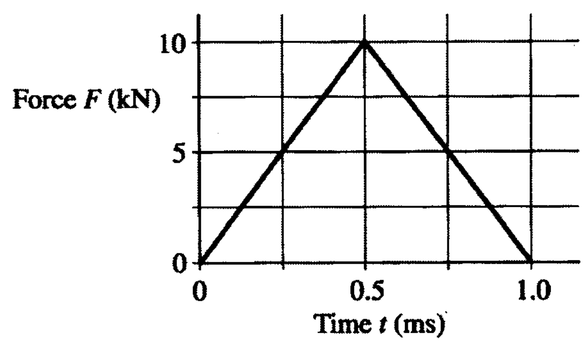
b) If the second ball had a mass of 2.4 kg, what was its speed after the collision?

$F\Delta t = m_2 \Delta v_2'$

$\Delta v_2' = \dfrac{F\Delta t}{m_2} = \dfrac{52}{2.4} = 21.7\ ms^{-1} = \boxed{22\ ms^{-1}}$

15. A 2.0 kg frictionless cart is moving at a constant speed of 3.0 ms^{-1} to the right on a horizontal surface, as shown, when it collides with a second cart of undetermined mass *m* that is initially at rest. The force *F* of the collision as a function of time t is shown in the graph below, where $t = 0$ is the instant of initial contact. As a result of the collision, the second cart acquires a speed of 1.6 ms^{-1} to the right. Assume that friction is negligible before, during, and after the collision.





Area $= (10 \times 10^3)(0.5 \times 10^{-3})$

$= \underline{5.0\ Ns}$.

a) Calculate the magnitude and direction of the velocity of the 2.0 cart after the collision.

Conservation of momentum:

$m_1 v_1 + \cancel{m_2 v_2} = m_1 v_1' + m_2 v_2'$

$v_1' = \dfrac{m_1 v_1 - m_2 v_2'}{m_1} = \dfrac{(2.0)(3.0) - (3.1)(1.6)}{2.0} = \boxed{+0.52\ ms^{-1}}$

(TO THE RIGHT)

b) Calculate the mass m of the second cart.

$F\Delta t =$ AREA UNDER GRAPH

$= m_2 \Delta v_2 \Rightarrow m_2 = \dfrac{\text{AREA}}{\Delta v_2} = \dfrac{5.0}{1.6} = \boxed{3.1\ kg}$