1.1 THE MATHEMATICS YOU NEED FOR IB PHYSICS

Notes

I. THE MATHEMATICS YOU NEED FOR IB PHYSICS

- A. ALGEBRA
- B. TRIGONOMETRY AND GEOMETRY
- C. WHAT ABOUT CALCULUS?
- D. PROBLEM-SOLVING

I. THE MATHEMATICS YOU NEED FOR IB PHYSICS

A. ALGEBRA

✓ Adding, subtracting, multiplying, dividing positive and negative numbers *If* m = 74.0, n = -92.5 and p = -18.5, find the following:

$$m + n$$
 $p - n$ $n - (-m)$ $-m(-n - p)$

✓ Working with exponents ('indexes') - whole and fractional, positive and negative

$$a^{-x} = \frac{1}{a^x}$$
 $\sqrt{a} = a^{1/2}$ $\sqrt[n]{a} = a^{1/n}$ $a^x a^y = a^{x+y}$ $(ab)^x = a^x b^x$

✓ Algebraically expanding and simplifying expressions

$$3(2x+5)$$
 $-ab(b-a)$ $2x(x^2-1)$

$$x(x^{2}+2x) - x^{2}(2-x)$$
 (2x+3)²

✓ Isolating variables and rearranging equations quickly and accurately

If v = u + at solve for a. If $v^2 = u^2 + 2as$ solve for u.

If
$$F = \frac{q_1 q_2}{4\pi \varepsilon_0 r^2}$$
 solve for r.

✓ Be able to estimate values of expressions without a calculator

$$\frac{243}{43}$$
 2.80 × 1.90 $\frac{312 \times 480}{160}$

$$\frac{8.99 \times 10^9 \times 7 \times 10^{-6} \times 7 \times 10^{-6}}{(8 \times 10^2)^2} \qquad \qquad \frac{6.6 \times 10^{-11} \times 6 \times 10^{24}}{(6.4 \times 10^6)^2}$$

✓ Given an equation, be able to say what happens to one variable when another changes

Given: y = cx, how does y change if x is tripled?

Given: $f = \frac{1}{2L} \sqrt{\frac{T}{\mu}}$, how does f change if T is doubled?

Given: PV = NkT, how does V change if P is doubled and T is halved?

✓ Adding, subtracting, multiplying, and dividing fractions with variables

Simplify the following:			
x 3x	а	6	3
$\frac{1}{4} + \frac{1}{5}$	$\frac{1}{3} + a$	<u> </u>	$-\frac{1}{4v}$

$$\frac{a}{2} \times \frac{a}{3} \qquad \qquad \left(\frac{4}{d}\right)^2 \qquad \qquad \frac{4}{x} \div \frac{x^2}{2}$$

✓ Finding slopes of lines (estimating and finding analytically)



✓ Solving systems of algebraic equations for 2 or more unknowns

Solve the system: 3x - 5y = 15y = 2x + 4

- ✓ Be comfortable leaving answers in terms of pi or some other constant, or even as a fraction
- \checkmark Being able to recognize even and odd functions and to sketch them quickly



Graphing data (recognizing independent and dependent variables) and interpreting graphs
 Graph the following experimental data and determine the relationship between them:

t(s)	s(m)	
0	0	
1	5	
2	20	
3	44	
4	78	
5	123	
6	176	

✓ Be able to convert from one set of units to another Convert 65 miles/hour to meters/second. ✓ Be able to look at a graph and see where the slope is positive, negative, zero, and how the slope is changing



✓ Recognizing that you can only add together homogeneous quantities (same type and units)

5kg = 3kg + 2kg $5kg \neq 3kg + 2m$ $5kg \neq 5ms^{-1}$

✓ Be familiar with logarithms, both natural and base-10

When
$$e^{y} = x$$
, $ln(x) = log_{e}(x) = y$ where $e = 2.71828183$
 $log_{b}(x y) = log_{b}(x) + log_{b}(y)$
 $logb(x/y) = logb(x) - logb(y)$
 $logb(x^{y}) = y \ log_{b}(x)$

✓ Be familiar with logarithmic graphs and scales, and recognize when to use them

Graph the following data:

X	У
0	0
1	0.01
2	0.1
3	1
4	10
5	100



✓ Finding maxima, minima and zeroes of functions is <u>helpful but not required</u>

B. TRIGONOMETRY AND GEOMETRY

✓ Making suitable scales to represent a physics situation in a diagram



Using right angle trigonometry – Pythagoras and basic trigonometric functions (sine, cosine, and tangent)



✓ Using the inverse trigonometric functions (sin⁻¹, cos⁻¹, and tan⁻¹) to find an angle *Find* θ *in each triangle:*



Being comfortable working with radians, and being able to convert between radians and degrees

One radian is the angle swept out by an arc of length equal to the radius of the circle.

radians =
$$\frac{\pi}{180} \times \text{degrees}$$



	0	π		
0	rad	1	0	rad
0			210	
30			225	
45			240	
60			270	
90			-	
120			300	
135			315	
150			330	
180			360	

degrees = $\frac{180}{100} \times radians$

✓ Recognizing and being able to graph the sine and cosine functions.



✓ Be really comfortable working with circles



$$C = 2\pi r = \pi d$$
 , Area = πr^2

Source: Mathematics for the iB Diploma, Oxford

✓ Finding areas and volumes of various shapes and solids (circles, spheres, squares, cones, triangles, cones, trapezoids, etc)

✓ Drawing accurate tangent lines to curves at a specific point



✓ Recognizing 30-60-90 and 45-45-90 triangles, and to solve them without a calculator

VALUES OF TRIGONOMETRIC
FUNCTIONS FOR COMMON ANGLES

θ	sin 0	cos θ	tan O
0°	0	1	0
30 °	1/2	√3/2	√3/3
37°	3/5	4/5	3/4
45°	$\sqrt{2}/2$	<u>√</u> 2/2	1
53°	4/5	3/5	4/3
60 °	√3/2	1/2	$\sqrt{3}$
90 °	1	0	∞

- ✓ Being able to express the relation between two quantities as a (unitless) ratio
- ✓ Knowing how to use trigonometric identities is <u>helpful but not required</u>
- ✓ Being able to solve non-right triangles (using law of sines, cosines, etc) is <u>helpful but not</u> required

C. WHAT ABOUT CALCULUS?

Calculus is not required for this course but it is more interesting (and at times easier!) with it. Many of the IB Physics texts refer to Calculus. You can always use calculus to solve an Physics problem but are never required to do so. Many of you will be learning Calculus while taking this course; if you are interested in Calculus-based explanations or derivations at any time, I am happy to give them to you.

Specifically useful would be a knowledge of (or interest in learning about):

- ✓ Working with the infinitely big (*limits*)
- ✓ Working with the infinitely small (*limits*)
- Understanding how the rate of change of a function over time is itself a function (*derivatives*)
- Understanding how the rate of change of the rate of change of a function over time is again a function (second derivatives)
- ✓ Being able to find the area between 2 non-linear functions or the area between a non-linear function and the x-axis (*integrals*)

D. PROBLEM-SOLVING

This is a critical part of IB Physics. On the IB exam, you will get marks for different parts of the problem - often you get a mark for simply writing down the correct formula! So following all of these steps is important. It also helps organize your work when things get complicated.

THE 5 STEPS FOR SOLVING PROBLEMS IN PHYSICS
STEP 1: Write down what you know and what you don't know.
STEP 2: Write down the appropriate formulae to solve the problem. Rearrange the formula if necessary and isolate the variable you wish to solve for.
STEP 3: Substitute into the formula the appropriate values you wrote down in step 1.
STEP 4: Perform the calculation. Always include proper units in your final answer.
STEP 5: Put a box around your final answer. Think about whether your answer makes sense .

Often in IB Physics, you need to go through this process more than once. For example, you may have to use the result of one problem to solve the next part of the problem. Also, you need to be able to find data for constants values in your data booklet (refer to it often!) Typically you will need to read a problem more than once to determine what it is asking.

Other tips for solving problems effectively in Physics:

- ✓ Use graph paper when possible (this makes it much easier to draw proper diagrams and graphs).
- ✓ Work in pencil, so you can easily erase any mistakes.
- ✓ Don't be afraid to use up space on the paper (don't cram your work into a small area).
- ✓ When working a problem in multiple steps (using the answer from one part to solve the next), generally you want to keep the same number of significant figures from one step to the next.
- ✓ Don't forget units and significant figures in your final answer.
- ✓ Know how to use your calculator.

EXAMPLE 1

When a stone is dropped off a cliff into the sea, the total distance fallen (d meters) is given by the formula $d = \frac{1}{2} g t^2$, where t is the time of fall in seconds and g is the gravitational constant 9.81 ms⁻². Find: a) the distance fallen after 4.0 seconds. **[79 m]**

b) the time taken for the stone to fall 200.0 meters. [6.39 s]

EXAMPLE 2

When a charge moves though a potential difference, a force acts upon it, and it is accelerated. By the conservation of energy, the change in potential energy (ΔPE) of the charge is equal to the change in kinetic energy (ΔKE).

a) Determine the speed of an electron accelerated through a potential difference of V = 2400 Volts. *Note:* $KE = \frac{1}{2} mv^2$ and PE = qV. [2.9 × 10⁷ ms⁻¹]

b) How far would this electron travel in 5.75 µs at this speed?

[167 m]