1.1 THE MATHEMATICS YOU NEED FOR IB PHYSICS — Notes

I. THE MATHEMATICS YOU NEED FOR IB PHYSICS

A. ALGEBRA

✓ **Adding, subtracting, multiplying, dividing positive and negative numbers**

If m = 74.0, n = -92.5 and p = -18.5, find the following:

$m + n$

$74.0 + {}^{-}92.5$

$= \boxed{-18.5}$

$p - n$

$-18.5 - {}^{-}92.5$

$= -18.5 + 92.5$

$= \boxed{74.0}$

$n - (-m)$

$= n + m$

$= -92.5 + 74.0$

$= \boxed{-18.5}$

$-m(-n-p)$

$= mn + mp$

$= -6845 + {}^{-}1369$

$= \boxed{-8214}$

OR

$-74.0(92.5 + 18.5)$

$= \boxed{-8214}$

✓ **Working with exponents ('indexes') - whole and fractional, positive and negative**

$$a^{-x} = \frac{1}{a^x} \qquad \sqrt{a} = a^{1/2} \qquad \sqrt[n]{a} = a^{1/n} \qquad a^x a^y = a^{x+y} \qquad (ab)^x = a^x b^x$$

✓ **Algebraically expanding and simplifying expressions**

$3(2x + 5)$

$\boxed{6x + 15}$

$-ab(b - a)$

$\boxed{-ab^2 + a^2b}$

$2x(x^2 - 1)$

$\boxed{2x^3 - 2x}$

$x(x^2 + 2x) - x^2(2 - x)$

$x^3 + 2x^2 - 2x^2 + x^3$

$= \boxed{2x^3}$

$(2x + 3)^2$

$(2x+3)(2x+3)$

$= 4x^2 + 6x + 6x + 9$

$= \boxed{4x^2 + 12x + 9}$

✓ **Isolating variables and rearranging equations quickly and accurately**

If $v = u + at$ solve for a.

$\boxed{a = \frac{v - u}{t}}$

If $v^2 = u^2 + 2as$ solve for u.

$\boxed{u = (v^2 - 2as)^{1/2}}$

If $F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$ solve for r.

$\boxed{r = \left(\frac{q_1 q_2}{4\pi\varepsilon_0 F}\right)^{1/2}}$

✓ **Be able to estimate values of expressions without a calculator**

$\frac{243}{43} \approx \frac{240}{40} = \boxed{6}$

$2.80 \times 1.90 \approx 3 \times 2 = \boxed{6}$

$\frac{312 \times 480}{160}$

$\approx \frac{300 \times 500}{200} = \frac{150000}{200} = \boxed{750}$

$\frac{8.99 \times 10^9 \times 7 \times 10^{-6} \times 7 \times 10^{-6}}{(8 \times 10^2)^2} \approx$

$\approx \frac{10 \times 50}{60} \times \frac{10^{-3}}{10^4} \approx \frac{500}{60} \times 10^{-7} \approx \boxed{8 \times 10^{-7}}$

$\frac{6.6 \times 10^{-11} \times 6 \times 10^{24}}{(6.4 \times 10^6)^2}$

$\approx \frac{40}{40} \times \frac{10^{13}}{10^{12}} = 1 \times 10^1 = \boxed{100}$

✓ **Given an equation, be able to say what happens to one variable when another changes**

Given: $y = cx$ *, how does y change if x is tripled?*

$y' = c(3x) = 3cx = 3y$ $\boxed{\therefore y \text{ is tripled}}$ $\Rightarrow \frac{y'}{y} = \frac{3cx}{cx} = 3.$

Given: $f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$ *, how does f change if T is doubled?*

$f' = \frac{1}{2L}\sqrt{\frac{2T}{\mu}} = \sqrt{2}\left(\frac{1}{2L}\sqrt{\frac{T}{\mu}}\right) = \sqrt{2}f$ $\boxed{\therefore f \text{ increased by a factor of } \sqrt{2}}$

$\frac{f'}{f} = \frac{\sqrt{2}f}{f} = \sqrt{2}$

Given: $PV = NkT$ *,how does V change if P is doubled and T is halved?*

$V = \frac{NkT}{P}$ $V' = \frac{Nk\frac{1}{2}T}{2P} = \frac{1}{4}\frac{NkT}{P} = \frac{1}{4}V.$

$\boxed{\therefore V \text{ decreases by factor of } \tfrac{1}{4}}$ $\frac{V'}{V} = \frac{1}{4}.$

✓ **Adding, subtracting, multiplying, and dividing fractions with variables**

Simplify the following:

$\frac{x}{4} + \frac{3x}{5}$

$\frac{5x}{20} + \frac{12x}{20} = \boxed{\frac{17x}{20}}$

$\frac{a}{3} + a$

$\frac{a}{3} + \frac{3a}{3} = \boxed{\frac{4a}{3}}$

$\frac{6}{y} - \frac{3}{4y}$

$\frac{24}{4y} - \frac{3}{4y} = \boxed{\frac{21}{4y}}$

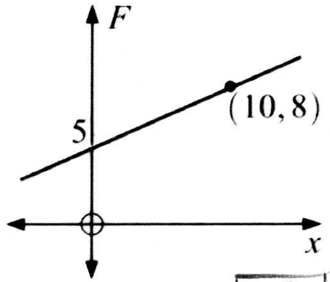
$\frac{a}{2} \times \frac{a}{3} = \boxed{\frac{a^2}{6}}$

$\left(\frac{4}{d}\right)^2 = \boxed{\frac{16}{d^2}}$

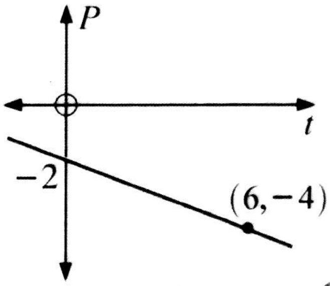
$\frac{4}{x} \div \frac{x^2}{2}$

$= \frac{4}{x} \cdot \frac{2}{x^2} = \boxed{\frac{8}{x^3}}$

✓ **Finding slopes of lines (estimating and finding analytically)**



$\text{SLOPE} = \frac{\Delta y}{\Delta x} = \boxed{\frac{3}{10}}$



$\text{SLOPE} = \frac{\Delta y}{\Delta x} = \frac{-2}{6} = \boxed{-\frac{1}{3}}$

✓ **Solving systems of algebraic equations for 2 or more unknowns**

Solve the system:

(1) $3x - 5y = 15$

(2) $y = 2x + 4$

Always check your answers!

METHOD 1

"Substitute" (2) into (1):

$3x - 5(2x+4) = 15$

$3x - 10x - 20 = 15$

$-7x = 35 \Rightarrow \boxed{x = -5}$

Now into (2):

$y = 2(-5) + 4 = -10 + 4 \Rightarrow \boxed{y = -6}$

METHOD 2

"elimination":

$3x - 5y = 15$

$3(-2x + y = -4)$

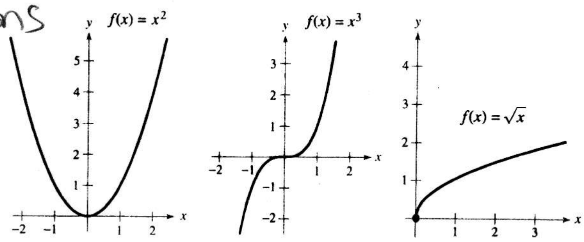
...etc...

✓ **Be comfortable leaving answers in terms of pi or some other constant, or even as a fraction**

✓ **Being able to recognize even and odd functions and to sketch them quickly**

EVEN functions have y-axis symmetry.

$y = x^2, x^4, x^6$, etc...



ODD functions have origin symmetry.

$y = x, x^3, x^5$, etc...

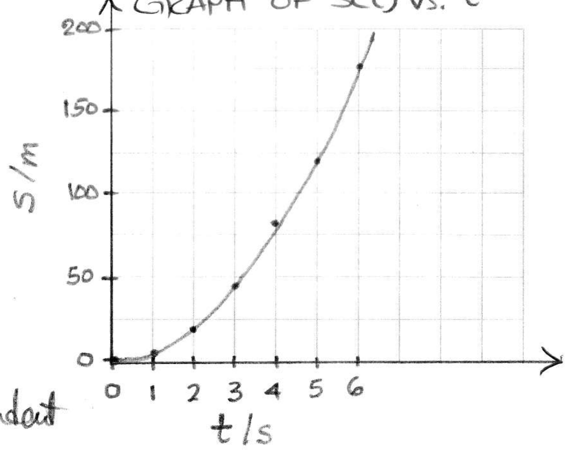
✓ **Graphing data (recognizing independent and dependent variables) and interpreting graphs**

Graph the following experimental data and determine the relationship between them:

t(s)	s(m)
0	0
1	5
2	20
3	44
4	78
5	123
6	176

independent (x) dependent (y)

GRAPH OF s(t) vs. t



CLEARLY NON-LINEAR, PROBABLY $s(t) \propto t^2$

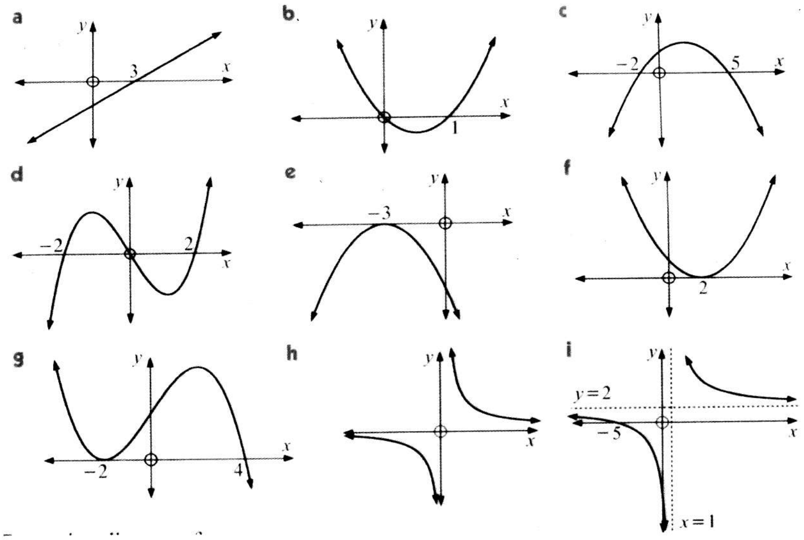
✓ **Be able to convert from one set of units to another**

Convert 65 miles/hour to meters/second.

$$\frac{65\,\cancel{mi}}{1\,\cancel{hr}} \times \frac{1\,\cancel{km}}{0.621\,\cancel{mi}} \times \frac{1000\,m}{1\,\cancel{km}} \times \frac{1\,\cancel{hr}}{3600\,s} = \frac{65 \times 1000}{0.621 \times 3600}$$

$$= \boxed{29\ m/s}$$

✓ **Be able to look at a graph and see where the slope is positive, negative, zero, and how the slope is changing**



✓ **Recognizing that you can only add together homogeneous quantities (same type and uni...**

5kg = 3kg + 2kg *5kg ≠ 3kg + 2m* *5kg ≠ 5ms⁻¹*

✓ **Be familiar with logarithms, both natural and base-10**

When $e^y = x$, $\ln(x) = \log_e(x) = y$ *where* $e = 2.71828183$

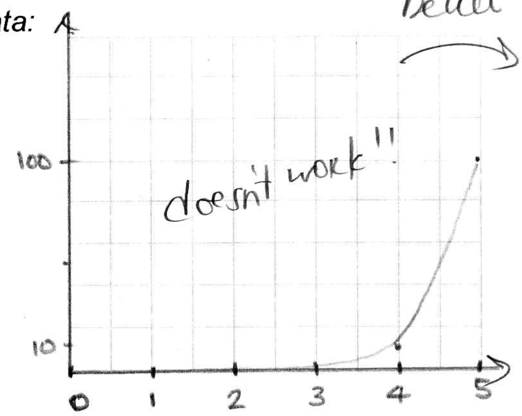
$\log_b(x\,y) = \log_b(x) + \log_b(y)$ $\log b(x/y) = \log b(x) - \log b(y)$ $\log b(x^y) = y \log_b(x)$

✓ **Be familiar with logarithmic graphs and scales, and recognize when to use them**

Graph the following data:

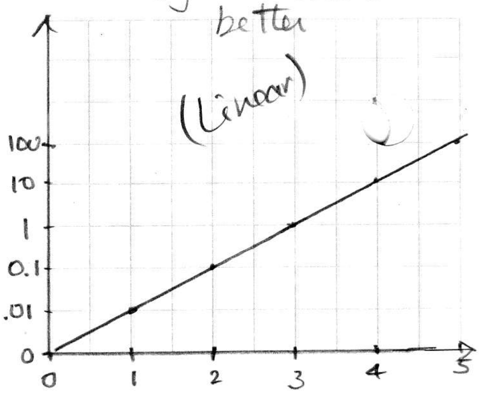
x	y
0	0
1	0.01
2	0.1
3	1
4	10
5	100

better



Log 10 scale is better

(Linear)



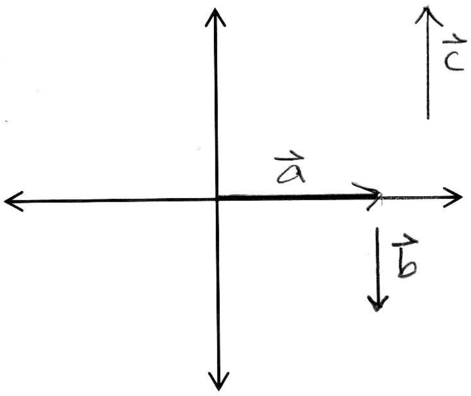
✓ **Finding maxima, minima and zeroes of functions is <u>helpful but not required</u>**

B. TRIGONOMETRY AND GEOMETRY

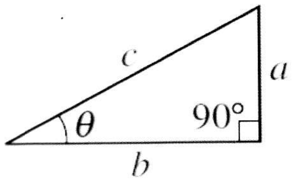
✓ **Making suitable scales to represent a physics situation in a diagram**

Graphically add the three vectors:
a *= 100 units east*
b *= 50 units south*
c *= 75 units north*

SCALE:
1 cm = 50 units



✓ **Using right angle trigonometry – Pythagoras and basic trigonometric functions (sine, cosine, and tangent)**



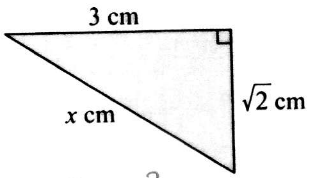
$$a^2 + b^2 = c^2$$

$$\sin\theta = \frac{a}{c}$$

$$\cos\theta = \frac{b}{c}$$

$$\tan\theta = \frac{a}{b}$$

Find x for each triangle:

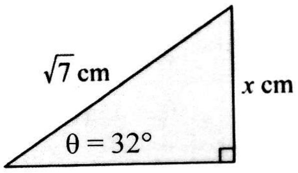


$a^2+b^2=c^2$

$c=(3^2+(\sqrt{2})^2)^{1/2}$

$=(9+2)^{1/2}$

$\boxed{c=\sqrt{11}}$



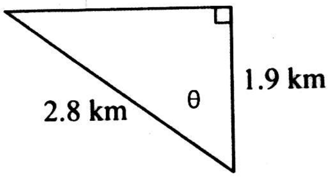
$\sin\theta = \frac{x}{\sqrt{7}}$

$x = \sqrt{7}\sin(32)$

$= \boxed{1.4\text{cm}}$

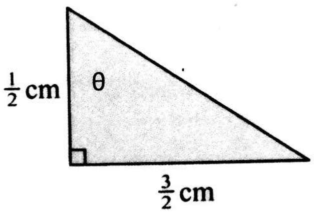
✓ **Using the inverse trigonometric functions ($\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$) to find an angle**

Find θ in each triangle:



$\cos\theta = \frac{1.9}{2.8}$

$\theta = \cos^{-1}\left(\frac{1.9}{2.8}\right) = \boxed{47°}$



$\tan\theta = \frac{\frac{3}{2}}{\frac{1}{2}} = 3$

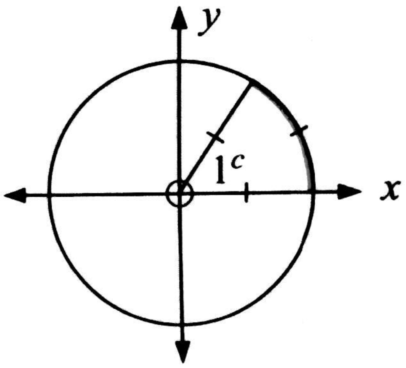
$\theta = \tan^{-1}(3)$

$= \boxed{72°}$

- ✓ **Being comfortable working with radians, and being able to convert between radians and degrees**

  One radian is the angle swept out by an arc of length equal to the radius of the circle.

$$\text{radians} = \frac{\pi}{180} \times \text{degrees} \qquad\qquad \text{degrees} = \frac{180}{\pi} \times \text{radians}$$

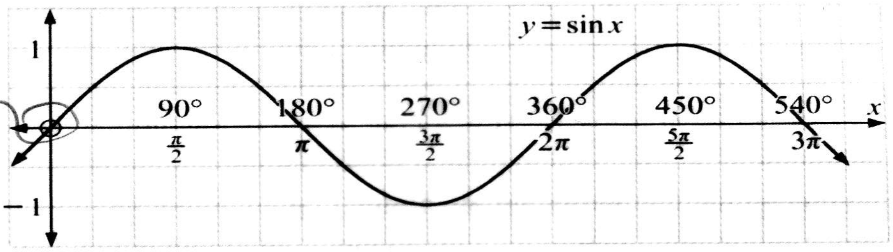


°	rad
0	0
30	$\pi/6$
45	$\pi/4$
60	$2\pi/6 = \pi/3$
90	$\pi/2$
120	$4\pi/6 = 2\pi/3$
135	$3\pi/4$
150	$5\pi/6$
180	π

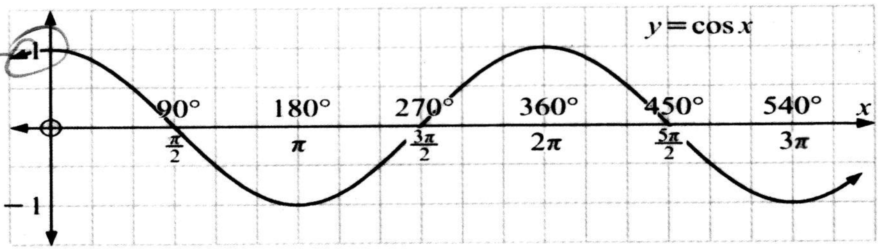
°	rad
210	$7\pi/6$
225	$5\pi/4$
240	$8\pi/6 = 4\pi/3$
270	$3\pi/2$
300	$10\pi/6 = 5\pi/3$
315	$7\pi/4$
330	$11\pi/6$
360	2π

- ✓ **Recognizing and being able to graph the sine and cosine functions.**

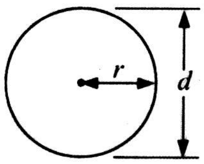
Relate to Right Δ's; Why 0?



Why 1



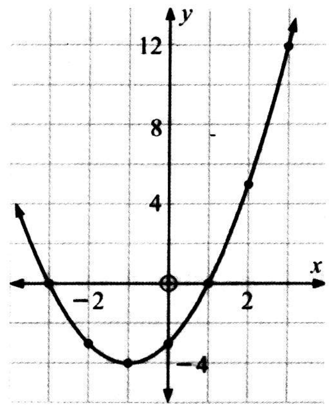
- ✓ **Be really comfortable working with circles**



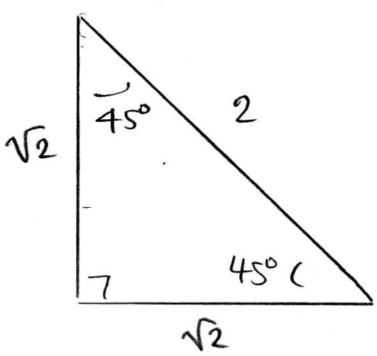
$$C = 2\pi r = \pi d \quad , \qquad \text{Area} = \pi r^2$$

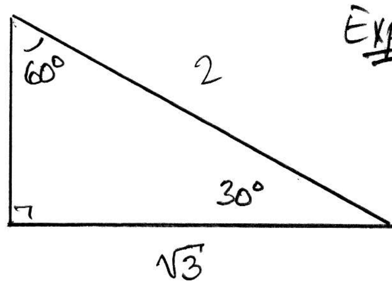
- ✓ **Finding areas and volumes of various shapes and solids (circles, spheres, squares, cones, triangles, cones, trapezoids, etc)**

✓ **Drawing accurate tangent lines to curves at a specific point**



✓ **Recognizing 30-60-90 and 45-45-90 triangles, and to solve them without a calculator**





THIS TABLE TELLS YOU THIS!

Explain

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	sin θ	cos θ	tan θ
0°	0	1	0
30°	1/2	√3/2	√3/3
37°	3/5	4/5	3/4
45°	√2/2	√2/2	1
53°	4/5	3/5	4/3
60°	√3/2	1/2	√3
90°	1	0	∞

✓ **Being able to express the relation between two quantities as a (unitless) ratio**

✓ **Knowing how to use trigonometric identities is <u>helpful but not required</u>**

✓ **Being able to solve non-right triangles (using law of sines, cosines, etc) is <u>helpful but not required</u>**

C. WHAT ABOUT CALCULUS?

Calculus is not required for this course *but it is more interesting (and at times easier!) with it. Many of the IB Physics texts refer to Calculus. You can always use calculus to solve an Physics problem but are never required to do so. Many of you will be learning Calculus while taking this course; if you are interested in Calculus-based explanations or derivations at any time, I am happy to give them to you.*

Specifically useful would be a knowledge of (or interest in learning about):

- ✓ **Working with the infinitely big (*limits*)**
- ✓ **Working with the infinitely small (*limits*)**
- ✓ **Understanding how the rate of change of a function over time is itself a function (*derivatives*)**
- ✓ **Understanding how the rate of change of the rate of change of a function over time is again a function (*second derivatives*)**
- ✓ **Being able to find the area between 2 non-linear functions or the area between a non-linear function and the x-axis (*integrals*)**

D. PROBLEM-SOLVING

This is a critical part of IB Physics. On the IB exam, you will get marks for different parts of the problem often you get a mark for simply writing down the correct formula! So following all of these steps is important. It also helps organize your work when things get complicated.

THE 5 STEPS FOR SOLVING PROBLEMS IN PHYSICS

STEP 1: **Read** the problem carefully (often you need to read it more than once to understand it) and draw a diagram if needed.

STEP 2: Keeping in mind the values given to you, **write** down the appropriate formula to use.

STEP 3: **Rearrange** the formula if necessary and **isolate** the variable you wish to solve for.

STEP 4: **Insert** all the values for variables you know (into the right side of the equation).

STEP 5: **Crunch** the numbers to get the answer (should be boxed, expressed in proper units and numbers of significant figures). Think about whether your answer makes sense.

Often in IB Physics, you need to go through this process more than once. For example, you may have to use the result of one problem to solve the next part of the problem. Also, you need to be able to find data for constants values in your data booklet (refer to it often!)

Other tips for solving problems effectively in Physics:

- ✓ Use graph paper when possible (this makes it much easier to draw proper diagrams and graphs)
- ✓ Work in pencil, so you can easily erase any mistakes
- ✓ Don't be afraid to use up space on the paper (don't cram your work into a small area)
- ✓ When working a problem in multiple steps (using the answer from one part to solve the next), generally you want to keep the same number of significant figures from one step to the next.
- ✓ Don't forget units and significant figures in your final answer
- ✓ **Know how to use your calculator**

EXAMPLES:

1. When a stone is dropped off a cliff into the sea, the total distance fallen (d meters) is given by th formula $d = \frac{1}{2} g t^2$, where t is the time of fall in seconds and g is the gravitational constant 9.81 ms^{-2}. Find:
   a. the distance fallen after 4.0 seconds.

   $$d = \frac{1}{2}gt^2 = \frac{1}{2}(9.81)(4.0)^2 = 78.48 = \boxed{79m}$$

   b. the time taken for the stone to fall 200.0 meters.

   SOLVE FOR t: $$t = \sqrt{\frac{2d}{g}} = \sqrt{\frac{2(200.0)}{9.81}} = \boxed{6.39s}$$

2. When a charge moves though a potential difference, a force acts upon it, and it is accelerated. By the conservation of energy, the change in potential energy (ΔPE) of the charge is equal to the change in kinetic energy (ΔKE).

   a. Determine the speed of an electron accelerated through a potential difference of V = 2400 Volts. *Note: $KE= \frac{1}{2} m v^2$ and $PE=qV$.*

Since $\Delta KE = \Delta PE$ (given)

$$\frac{1}{2} m v^2 = qV$$

$$v = \sqrt{\frac{2qV}{m}} = \left(\frac{2(1.6\times10^{-19})(2400)}{9.11\times10^{-31}}\right)^{1/2} = \boxed{2.9\times10^{7}\ m/s}$$

   b. How far would this electron travel in 5.75 μs at this speed?

$$v = \frac{s}{t} \Rightarrow s = vt = (2.903\times10^{7})(5.75\times10^{-6})$$

$$= 166.95$$

$$= \boxed{167\ m}$$