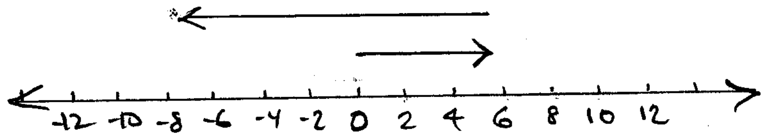
2.1 KINEMATICS – Solutions to Class Notes examples

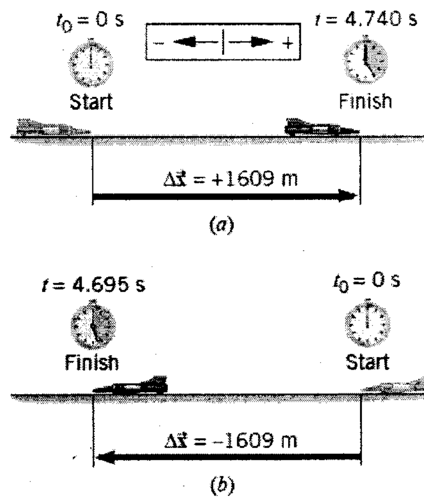
EXAMPLE 1
A mass initially at O first moves 5 m to the right and then 12 m to the left. What is the total distance covered by the mass and what is its change in displacement? **[17 m, -7 m]**



TOTAL $s = 5m + 12m$
$= \boxed{17m}$

$\Delta\vec{s} = \vec{s}_f - \vec{s}_i$
$= -7 - 0 = \boxed{-7m}$

EXAMPLE 2
A world land speed record was set in 1997 in a car powered by 2 jet engines. Its aim was to exceed the speed of sound (about 343 ms^{-1}). To establish this record, the driver has to make 2 runs through the course in each direction to nullify wind effects. The final value for the established speed is then the average of the two speeds.



Source: Physics, 8th Ed, Cutnell & Johnson

a) From the diagram, determine the average velocities for both trips. **[+339.5 ms^{-1}, -342.7 ms^{-1}]**

(a) $\vec{v}_{AVE} = \frac{\Delta\vec{s}}{\Delta t} = \frac{+1609m}{4.740s} = \boxed{+339.5\ ms^{-1}}$

(b) $\vec{v}_{AVE} = \frac{\Delta\vec{s}}{\Delta t} = \frac{-1609m}{4.695s} = \boxed{342.7\ ms^{-1}}$

b) Did the driver exceed the speed of sound?

AVE SPEED $= \frac{339.5 + 342.7}{2} = \boxed{341.1\ ms^{-1}}$ Nope!

c) In what direction was the wind probably blowing?

RIGHT TO LEFT, BECAUSE IT TOOK LESS TIME TO COVER THE DISTANCE GOING THAT WAY.

EXAMPLE 3
A car starts out from O in a straight line and moves a distance of 20.0 km towards the right, and then returns to its starting position 1.0 hr later. What is the average speed and the average velocity for this trip?
[40 kmhr^{-1}, 0 ms^{-1}]

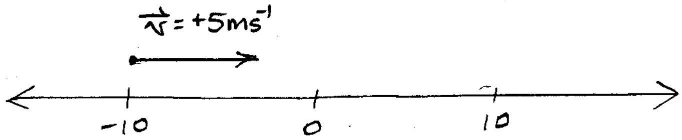
$$v_{AVE} = \frac{\Delta s}{\Delta t} = \frac{40.0}{1.0} = \boxed{40\ km\,hr^{-1}}$$

$\vec{v}_{AVE} = \boxed{0}$ because the car ended where it started ($\Delta \vec{s} = 0$).

EXAMPLE 4
A car moves in exactly the same way as in example 3, but this time it starts out not at O but at a point 100.0 km to the right of O. What is the average speed and average velocity for this trip? **[40 kmhr^{-1}, 0 ms^{-1}]**

SAME ANSWERS.
Where it starts does not matter.

EXAMPLE 5
The initial displacement of a body moving with a constant velocity of 5 ms^{-1} is -10 m. When does the body reach the point with displacement 10 m? What distance does the body cover in this time? **[4 s, 20 m]**



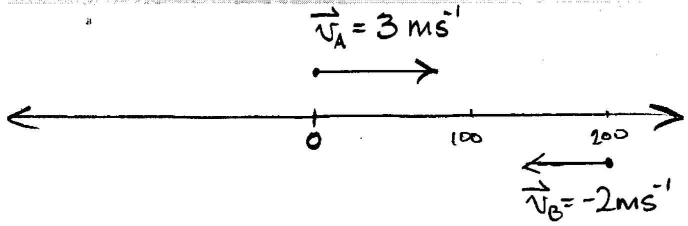
TOTAL DISTANCE TRAVELLED IS $\boxed{+20\,m.}$

So, $v = \frac{\Delta s}{\Delta t} \Rightarrow \Delta t = \frac{\Delta s}{v}$

$$= \frac{20}{5}$$

$$= \boxed{4\,s}$$

EXAMPLE 6
Bicyclist A starts with initial displacement zero and moves with velocity 3 ms^{-1}. At the same time, bicyclist B starts from a point with displacement 200 m and moves with a velocity -2 ms^{-1}. When does A meet B, and where are they when this happens?
[40 s, 120 m]



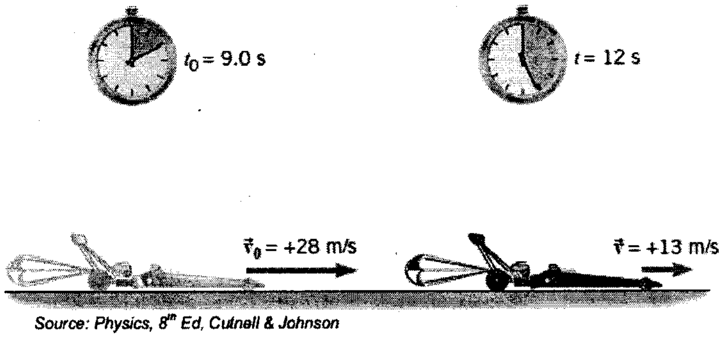
$v_A = \frac{\Delta s_A}{\Delta t_A}$ $\quad v_B = \frac{\Delta s_B}{\Delta t_B} = \frac{200 - \Delta s_A}{\Delta t_B}$

EQUATE Δs_A & Δs_B AND NOTE THAT $\Delta t_A = \Delta t_B$.

$\Delta s_A = v_A \Delta t_A$; $\quad \Delta s_B = v_B \Delta t_B$

EXAMPLE 7

A drag racer crosses the finish line, and the driver deploys a parachute and applies the brakes to slow down, as shown. The driver begins slowing down at t = 9.0 s when **v** = +28 ms^{-1}. When t = 12.0 s, **v** = +13 ms^{-1}. What was the acceleration of the dragster? **[-5.0 ms^{-1}]**



Source: Physics, 8th Ed, Cutnell & Johnson

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$
$$= \frac{13 - 28}{12.0 - 9.0}$$
$$= \boxed{-5.0\ \text{ms}^{-1}}$$

EXAMPLE 8

Consider the following s-t graph.
Determine:

a) v_{ave} between 0 and 2 s **[1 ms^{-1}]**

$v_{AVE} = \frac{\Delta s}{\Delta t} = \frac{2-0}{2-0} = \boxed{1\ \text{ms}^{-1}}$

b) v_{ave} between 2 s and 4 s **[3 ms^{-1}]**

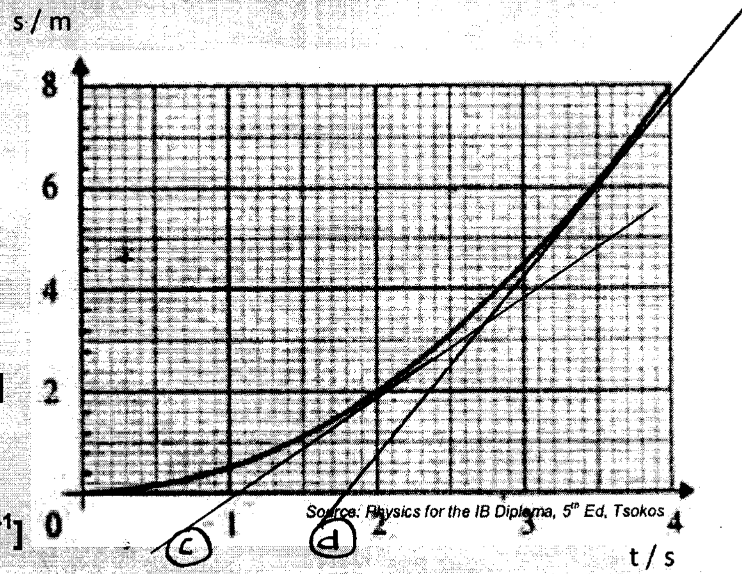
$v_{AVE} = \frac{\Delta s}{\Delta t} = \frac{8-2}{4-2} = \boxed{3\ \text{ms}^{-1}}$

c) v at 2 s **[2 ms^{-1}]**

SLOPE OF LINE c $\approx \boxed{2\ \text{ms}^{-1}}$

d) v at 3.5 s **[~3.8 ms^{-1}]**

SLOPE OF LINE d $\approx \boxed{3.8\ \text{ms}^{-1}}$



Source: Physics for the IB Diploma, 5th Ed, Tsokos

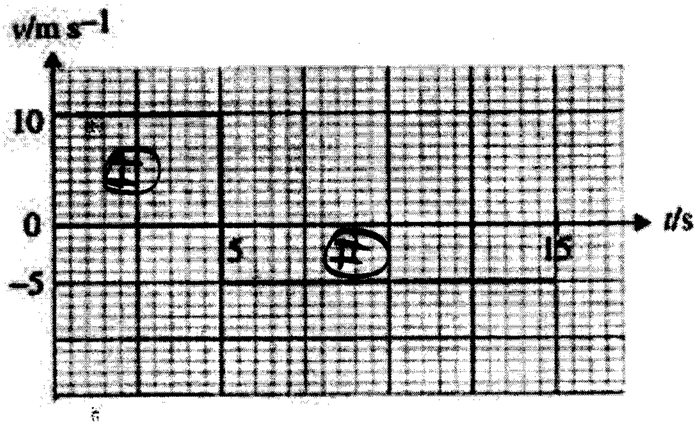
e) v_{ave} between 0 s and 4 s **[2 ms^{-1}]**

$v_{AVE} = \frac{\Delta s}{\Delta t} = \frac{8-0}{4-0} = \boxed{2\ \text{ms}^{-1}}$

EXAMPLE 9

A mass starts out from O with a velocity +10 ms^{-1} and continues moving at this velocity for 5 s. The velocity is then abruptly reversed to -5 ms^{-1} and the object moves at this velocity for 10 s. For this motion, find:

a) the change in displacement [0 m]
b) the total distance travelled [100 m]
c) the average speed [6.7 ms^{-1}]
d) the average velocity [0 ms^{-1}]



a) $\Delta \vec{s}$ = AREA BETWEEN GRAPH AND X-AXIS;

AREA (I) $= (+10)(5) = +50$ m.

AREA (II) $= (-5)(10) = -50$ m $\therefore$ AREA(I) + AREA(II) $= \boxed{0\text{ m}}$.

b) PART (I) : $v = \frac{s}{t} \Rightarrow s_I = vt = (10)(5) = 50$ m.

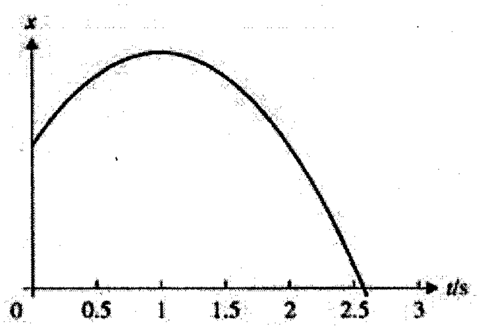
PART (II) : $v = \frac{s}{t} \Rightarrow s_{II} = vt = (5)(10) = 50$ m.

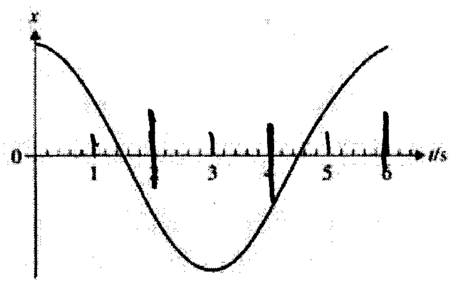
$s_I + s_{II} = 50 + 50 = \boxed{100\text{ m}}$

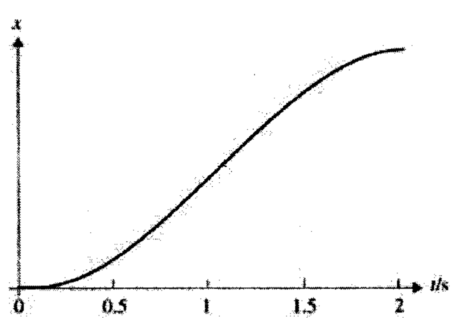
c) $v_{AVE} = \frac{\Delta s}{\Delta t} = \frac{100}{15} = \boxed{6.7\text{ ms}^{-1}}$.

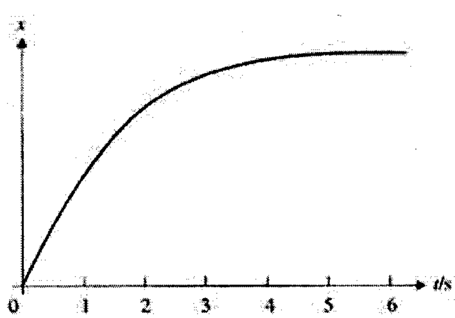
d) $\vec{v}_{AVE} = \frac{\Delta \vec{s}}{\Delta t} = \frac{0}{15} = \boxed{0\text{ ms}^{-1}}$

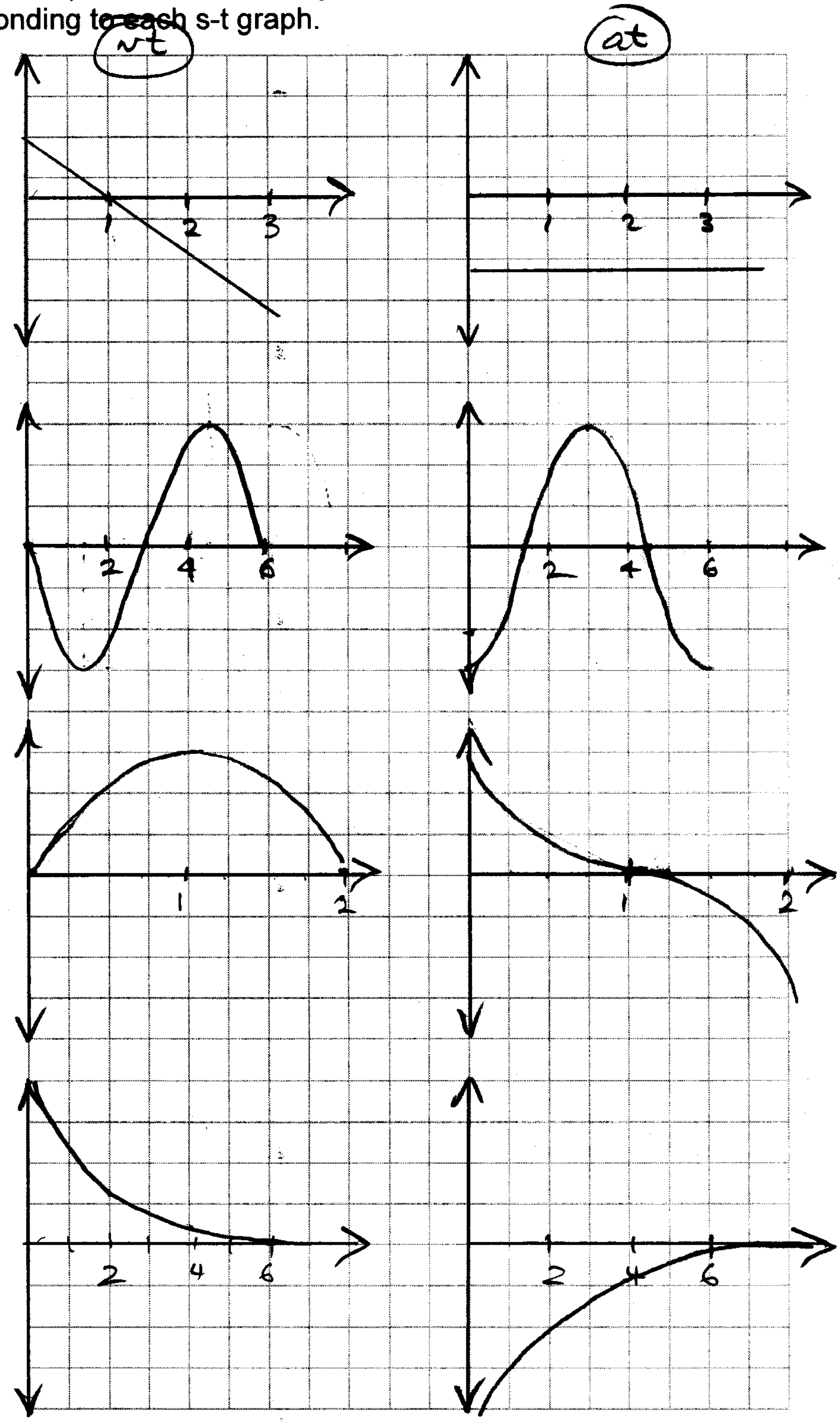
The graphs show the variation of the displacement of an object with time (s-t graphs). Sketch a v-t graph and an a-t graph corresponding ~~to each~~ s-t graph.





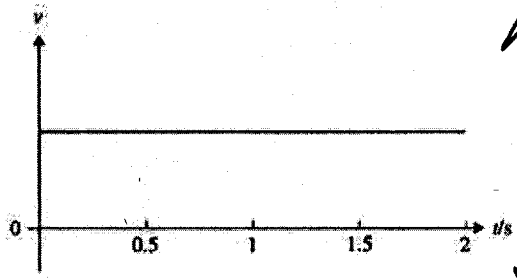


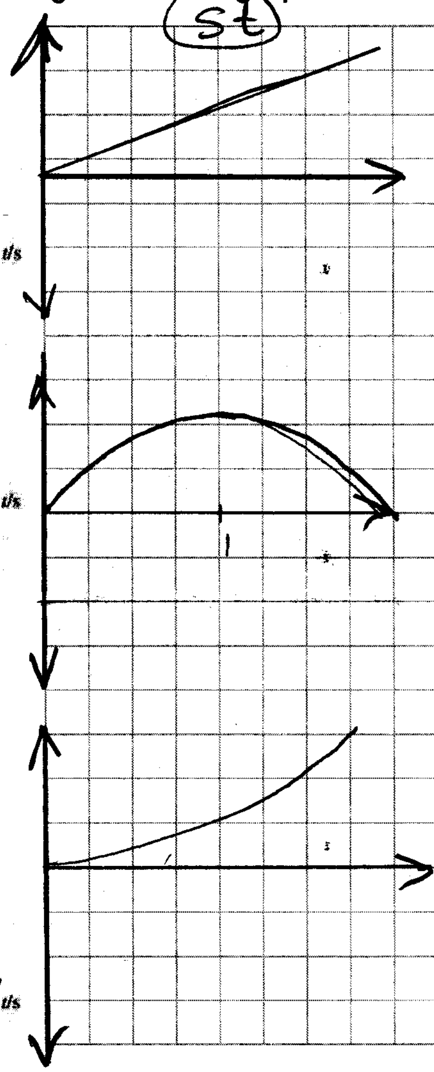


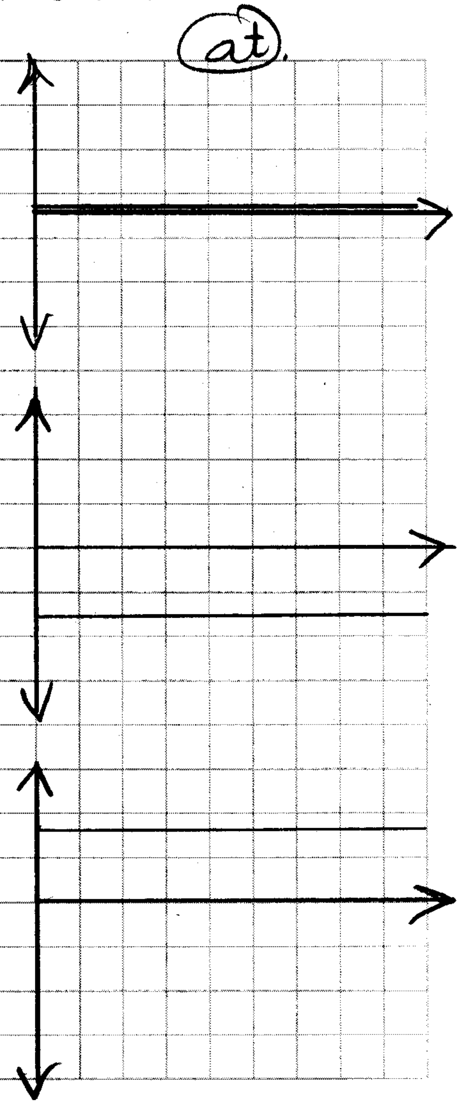


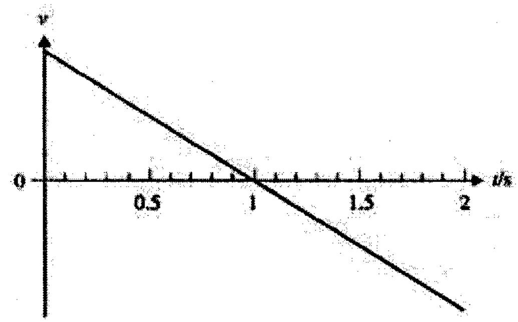
Source: Physics for the IB Diploma, 5th Ed, Tsokos

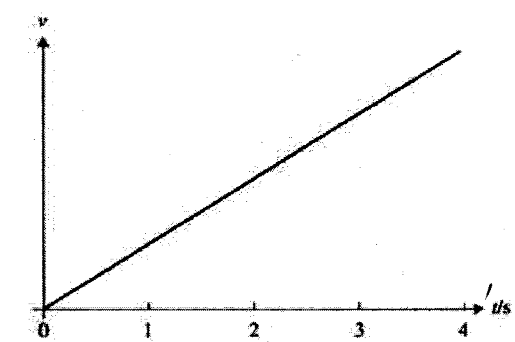
The graphs show the variation of the velocity of an object with time (v-t graphs). Sketch an s-t graph and an a-t graph corresponding to each ~~v-t~~ graph.











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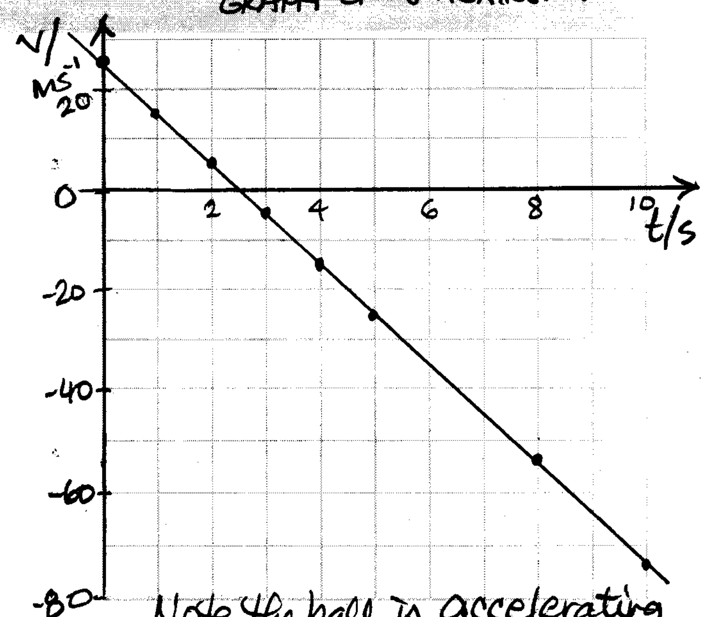
EXAMPLE 10
An cannonball initially at a height of 1.0 m in a cannon on the edge of a high cliff is shot directly upwards with an initial velocity of 25.0 ms^{-2}.

a) Fill in the following table and graph the data:

Time / s	Velocity / ms^{-1}
0.0	+25.0
1.0	+15.2
2.0	+5.4
3.0	-4.4
4.0	-14.2
5.0	-24.1
8.0	-53.5
10.0	-73.1

GRAPH OF v AGAINST t



Note the ball is accelerating downwards even when moving upwards! :)

b) From your graph, what is the acceleration of the ball?

$$SLOPE = \frac{\Delta y}{\Delta x} = \frac{\Delta v}{\Delta t} = \frac{-73.1-25.0}{10.0-0} = \boxed{9.81\ ms^{-2}} \ !!!$$

(using all data)

EXAMPLE 10
A mass has initial velocity of 10.0 ms^{-1}. It moves with an acceleration of -2.00 ms^{-2}. When will it have zero velocity? **[5.00 s]**

s	u	v	a	t
	10.0	0	-2.00	?

$v = u + at$

$$\Rightarrow t = \frac{v-u}{a} = \frac{0-10.0}{-2.00}$$

$$= \boxed{5.00\ s}$$

EXAMPLE 11
What is the displacement after 10.0 s of a mass whose initial velocity is 2.00 ms^{-1} and moves with an acceleration of 4.00 ms^{-2}? **[+220 m]**

s	u	v	a	t
?	2.00		4.00	10.0

$$s = ut + \tfrac{1}{2}at^2$$
$$= (2.00)(10.0) + \tfrac{1}{2}(4.00)(10.0)^2$$
$$= 20.0 + \tfrac{1}{2}(400)$$
$$= \boxed{+220\ m}$$

EXAMPLE 12
A car has initial velocity of 5.0 ms^{-1}. When its displacement increases by 20.0 m, its velocity becomes 7.0 ms^{-1}. What is the acceleration? **[0.60 ms^{-2}]**

s	u	v	a	t
+20.0	+5.0	+7.0	?	

$v^2 = u^2 + 2as$

$\Rightarrow a = \frac{v^2 - u^2}{2s} = \frac{(7.0)^2 - (5.0)^2}{2(20.0)}$

$= \frac{49-25}{40.0} = \boxed{0.60\ ms^{-2}}$

EXAMPLE 13
A body has initial velocity of 4.0 ms^{-1} and a velocity of 12 ms^{-1} after 6.0 s. What displacement did the body cover in the 6.0 s? **[48 m]**

s	u	v	a	t
?	4.0	12		6.0

$s = \frac{t}{2}(v+u) = \frac{6.0}{2}(4.0+12)$

$= 3.0(16)$

$= \boxed{48m}$

EXAMPLE 14
Two balls start out moving to the right with constant velocities of 5 ms^{-1} and 4 ms-1. The slow ball starts first and the other 4 s later. How far from the starting position are they when they meet? **[+80 m]**

SLOW BALL:

s_1	u	v	a	t
?	4	4		t

FAST BALL:

s_2	u	v	a	t
?	5	5		t-4

the two displacements are equal when they meet ($s_1 = s_2$).

$s = \frac{t}{2}(v+u)$ and thus:

$\Rightarrow \frac{t-4}{2}(5+5) = \frac{t}{2}(4+4)$

solve for t...

$5(t-4) = 4t$

$5t - 20 = 4t$

$t = 20s$ they meet.

then, $s = \frac{t}{2}(v+u)$

$= \frac{20}{2}(4+4) = \boxed{+80m}$

EXAMPLE 15
A mass is thrown upwards with an initial velocity of 30 ms^{-1}. A second mass is dropped from directly above, a height of 60 m from the first mass, 0.5 s later. When do the masses meet and how high is the point where they meet? [t = 2.34s, **[+42.9 m]**

MASS #1

s_1	u	v	a	t
s_1	+30		-9.81	t_1

MASS #2

s_2	u	v	a	t
s_1+60	0		-9.81	t_1-0.5

they meet when $s_1 = s_2$

$s = ut + \frac{1}{2}at^2$ and thus

$\overbrace{30t + \frac{1}{2}(9.81)t^2}^{s_1} = \overbrace{60 + \frac{1}{2}(-9.81)(t-0.5)^2}^{s_2}$

Solve for t...

$30t + {-4.9}t^2 = 60 + {-4.9}(t-0.5)^2$

$= -4.9(t^2 - t + .25) + 60$

$30t - \cancel{4.9t^2} = -4.9t^2 + 4.9t - 1.2 + 60$

$25.1t = 58.8 \Rightarrow \underline{t = 2.34s.}$

then,

$s = ut + \frac{1}{2}at^2$

$= 30(2.34) + \frac{1}{2}(-9.81)(2.34)^2 = \boxed{43.3m}$

EXAMPLE 16

A ball is dropped from rest from a height of 20.0 m. One second later a second ball is thrown vertically downwards. If the two balls arrive on the ground at the same time, what must have been the initial velocity of the second ball? **[-15 ms^{-1}]**

BALL #1

s_1	u_1	v_1	a	t_1
-20.0	0		-9.81	t_1

Solve for t_1:

$s = \cancel{ut} + \frac{1}{2}at^2$

$\Rightarrow t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2(20.0)}{9.81}} = \underline{2.02\,s}$

BALL #2

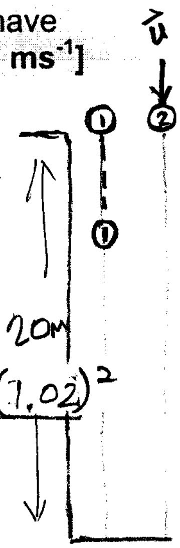
s_2	u_2	v_2	a	t_2
-20.0	?		-9.81	$t_1 - 1$

then find u_2 using this t_1:

$s = ut + \frac{1}{2}at^2$

$\Rightarrow u = \frac{s - \frac{1}{2}at^2}{t} = \frac{(-20.0) - \frac{1}{2}(-9.81)(1.02)^2}{1.02}$

$\boxed{-14.6\ ms^{-1}}$



EXAMPLE 17

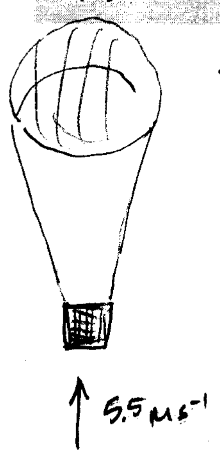
A hot air balloon is rising vertically at a constant speed of 5.0 ms^{-1}. A sandbag is released and it hits the ground 12.0 s later. (Assume balloon starts on ground).

a) With what speed does the sandbag hit the ground? **[115 ms^{-1}]**

b) How high was the balloon when the sandbag was released? **[660 m]**

* c) What is the relative velocity of the sandbag with respect to the balloon 6.0 s after it was dropped? **[-60.5 ms^{-1}]**

Assume the balloon's velocity increased to 5.5 ms^{-1} after releasing the sandbag. Draw a picture to help you visualize this question.



SANDBAG:

s	u	v	a	t
?	+5.0	?	-9.81	12.0

$v = u + at = +5.0 + (-9.81)(12.0)$

$\boxed{v = -113\ ms^{-1}}$ (a)

Now find s:

$s = \frac{t}{2}(v+u) = \frac{12.0}{2}(-113 + 5.0) = \boxed{650m}$

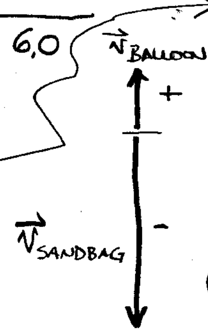
FOR PART (c):

SANDBAG:

s	u	v	a	t
	+5.0	?	-9.81	6.0

$v = u + at = +5.0 + (-9.81)(6.0)$

$= -53.9\ ms^{-1}$



since balloon has velocity $+5.5\ ms^{-1}$ at $t = 6.0\ s$ (upward) it follows that the relative velocity between them is

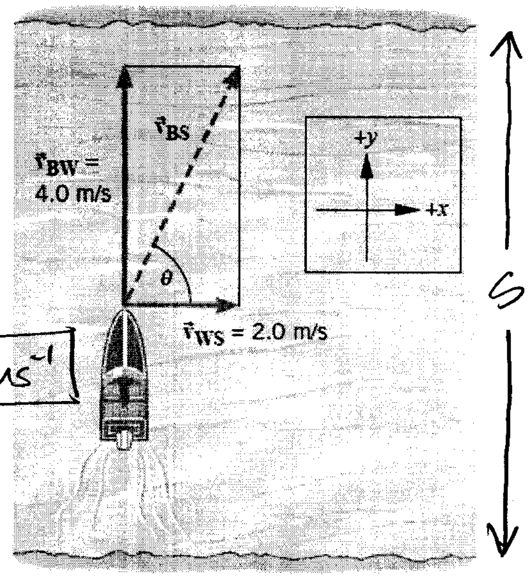
$5.5 + 53.9 = \boxed{59.4\ ms^{-1}}$

(they are moving away from one another at this rate). ☺

EXAMPLE 18

The engine of a boat drives it across a river that is 1800 m wide. The velocity $\mathbf{v}_{BW}$ of the boat relative to the water is 4.0 ms^{-1}, directed perpendicular to the current, as shown. The velocity $\mathbf{v}_{WS}$ of the water relative to the shore is 2.0 ms^{-1}.

a) What is the velocity $\mathbf{v}_{BS}$ of the boat relative to the shore? **[4.5 ms^{-1} at 63°]**



$$v_{BS}^2 = v_{BW}^2 + v_{WS}^2$$

$$v_{BS} = \sqrt{v_{BW}^2 + v_{WS}^2} = \sqrt{4.0^2 + 2.0^2} = \boxed{4.5\ ms^{-1}}$$

$$\theta = \tan^{-1}\left(\frac{v_{BW}}{v_{WS}}\right) = \tan^{-1}\left(\frac{4.0}{2.0}\right) = \boxed{63^\circ}$$

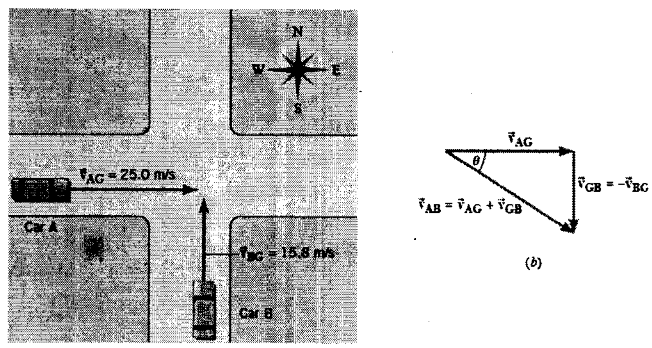
b) How long does it take for the boat to cross the river? **[450 s]**

$$v_{BW} = \frac{s}{t} \Rightarrow t = \frac{s}{v_{BW}} = \frac{1800}{4.0} = \boxed{450\ s}$$

NOTE: v_{WS} is irrelevant!

EXAMPLE 19

Find $\mathbf{v}_{AB}$, the velocity of car A as measured by a passenger in car B. **[29.6 ms^{-1} at θ=32.4°]**



(a) (b)

Source: Physics, 8th Ed, Cutnell & Johnson

$$v_{AB}^2 = v_{AG}^2 + v_{GB}^2$$

$$v_{AB} = \sqrt{v_{AG}^2 + v_{GB}^2}$$

$$= \sqrt{25.0^2 + 15.8^2} = \boxed{29.6\ ms^{-1}}$$

$$\theta = \tan^{-1}\left(\frac{v_{GB}}{v_{AG}}\right) = \tan^{-1}\left(\frac{15.8}{25.0}\right)$$

$$= \boxed{32.3^\circ}$$