Topic 1

Where appropriate, $1 \checkmark = 1$ mark

- **1** B
- 2 A
- 3 D
- **4** B
- 5 A
- 6 D
- 7 C
- 8 A
- 9 C
- **,** C
- 10 A
- 11 Use a smaller heavier ball. 🗸

In order to minimise the effect of air resistance. \checkmark

Let the ball drop from various heights. \checkmark

In order to draw a graph of height versus time and get the acceleration through the gradient of the graph. \checkmark If a stopwatch is to be used measure the time for each height many times and get an average. \checkmark In order to get a more accurate value for the time. \checkmark

12 a It will take $\frac{30}{4.0} = 7.5$ s to get across.

And he will move $3.0 \times 7.5 = 22.5 \approx 22$ m to the right of P. \checkmark

b Correct diagram. ✓

$$\sin \theta = \frac{3.0}{4.0} = 0.75 \checkmark$$

$$\theta = \sin^{-1} 0.75 = 48.6^{\circ} \checkmark$$

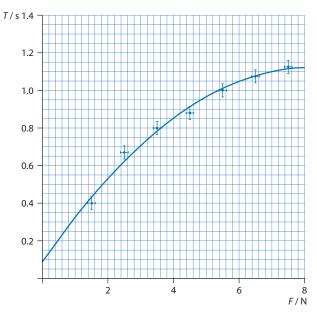
$$4.0 \text{ m s}^{-1}$$

c The woman moves across with a speed of $\sqrt{4.0^2 - 3.0^2} = 2.6458 \text{ m s}^{-1}$.

So she will take a time of $\frac{30}{2.6458} = 11.3 \approx 11 \text{ s}$, so will be longer than the man. \checkmark

13 a Smooth curve. \checkmark

Through all the error bars. \checkmark



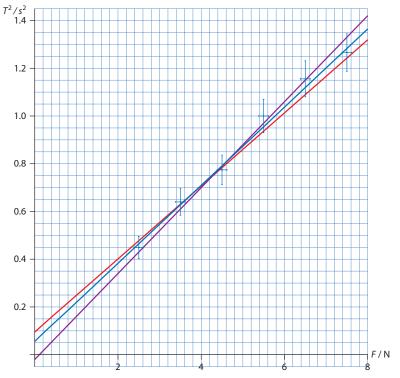
- **b** The vertical intercept is about 0.1 s. \checkmark
- **c** For *T* to be proportional to *F* requires a straight line graph through the origin. \checkmark And here neither of these conditions are satisfied. \checkmark
- **d** The uncertainty in *T* is about ± 0.035 s. \checkmark

$$\frac{\Delta T^2}{T^2} = 2\frac{\Delta T}{T} \Rightarrow \Delta T^2 = 2T\Delta T \checkmark$$

Hence $\Delta T^2 = \pm 2 \times 1.0 \times 0.035 = \pm 0.07 \text{ s}^2 \checkmark$

e Correct plotting of points. ✓

Correct error bars and lines of maximum and minimum slope. ✓ Line of best-fit is straight and within uncertainties passes through origin. ✓ Hence claim is correct. ✓



f Slope of line of best fit 0.164 s² N⁻¹. ✓ Max/min slopes 0.153 s² N⁻¹ and 0.180 s² N⁻¹ so uncertainty is 0.0135 ≈ 0.01 s² N⁻¹. ✓ So (0.164 ± 0.001) s² N⁻¹. ✓

Topic 10

Where appropriate, $1 \checkmark = 1$ mark

- C
 C
 C
 C
 C
 C
 C
 C
 C
 C
 T
 D
- 8 B
- 9 C
- 10 A

11 a The potential at the surface is $V = -\frac{GM}{R} = -5.0 \times 10^{12} \text{ J kg}^{-1}$.

And so
$$M = -\frac{VR}{G} = \frac{5.0 \times 10^{12} \times 2.0 \times 10^5}{6.67 \times 10^{-11}} = 1.5 \times 10^{28} \text{ kg.}$$

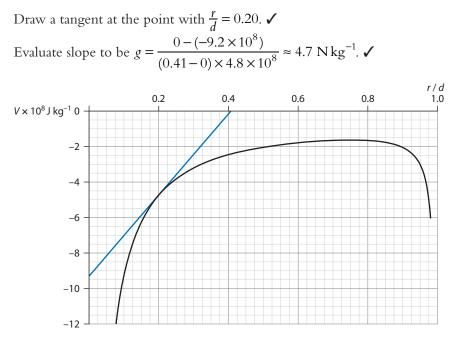
b The potential energy at launch on the surface of the planet is mV.

And so the total energy at launch is $\frac{1}{2}mv^2 + mV$.

At the escape speed the total energy has to be zero. \checkmark

- And the result follows. **c** $v = \sqrt{-2V} = \sqrt{2 \times 5.0 \times 10^{12}}$
 - Which equals $v = 3.2 \times 10^6 \text{ m s}^{-1}$.
- **d** The work required is $W = m\Delta V$ with $\Delta V = (-1.2 \times 10^{12} (-5.0 \times 10^{12}) = 3.8 \times 10^{12} \text{ J kg}^{-1}$. And this is $W = 1500 \times 3.2 \times 10^{12} = 5.7 \times 10^{15} \text{ J}$.
- e The additional energy needed is the kinetic energy: from $\frac{mv^2}{r} = \frac{GMm}{r^2}$ we find $E_{\rm K} = \frac{1}{2}\frac{GMm}{r} = -\frac{1}{2}mV$ where *V* is the potential at the position of the probe. \checkmark And this is $E_{\rm K} = -\frac{1}{2} \times 1500 \times (-1.2 \times 10^{12}) = 9.0 \times 10^{14}$ J. \checkmark

f The potential at the release point is $V_1 = -2.2 \times 10^{12} \text{ J kg}^{-1}$ and from conservation of energy $mV_1 = mV_2 + \frac{1}{2}mv^2$ where is the potential at the surface. \checkmark Hence $v = \sqrt{2(V_1 - V_2)} = \sqrt{2(-2.2 \times 10^{12} - (-5.0 \times 10^{12}))} = 2.4 \times 10^6 \text{ m s}^{-1}$. 12 a The slope of the tangent is gravitational field strength. \checkmark



b The gravitational potential has zero slope there. ✓
 Which implies that the gravitational field strength is zero at that point. ✓

c
$$g = \frac{GM}{r_1^2} - \frac{Gm}{r_2^2} \checkmark$$

 $0 = \frac{GM}{0.75^2} - \frac{Gm}{0.25^2} \checkmark$
Giving $\frac{M}{m} = \frac{0.75^2}{0.25^2} = 9.0 \checkmark$
13 a $qV_1 + \frac{1}{2}mv^2 = qV_2$ i.e. $q\frac{kQ}{r_1} + \frac{1}{2}mv^2 = q\frac{kQ}{r_2} \checkmark$
 $2.4 \times 10^{-6} \times \frac{8.99 \times 10^9 \times 8.8 \times 10^{-6}}{0.75} + \frac{1}{2} \times 0.0075 \times 3.2^2 = 2.4 \times 10^{-6} \times \frac{8.99 \times 10^9 \times 8.8 \times 10^{-6}}{r_2} \checkmark$
 $0.2532 + 0.3840 (= 0.6372) = \frac{0.1899}{r_2}$
Hence $r_2 = 0.2980 \approx 0.30$ m. ✓

- b The pellet will move radially away from the sphere. ✓
 With an increasing speed but a decreasing acceleration. ✓
- c The total energy of the pellet is 0.6372 J and far away this will turn into kinetic energy. \checkmark

Hence
$$\frac{1}{2} \times 0.075 \times v^2 = 0.6372$$
 J leading to 4.1 m s⁻¹.

14 a
$$qV = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2qV}{m}} \checkmark$$

Hence $v = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 29.1}{9.11 \times 10^{-31}}} = 3.197 \times 10^6 \approx 3.2 \times 10^6 \text{ m s}^{-1}. \checkmark$

b The horizontal distance of 2.0 cm is covered at the constant speed found above. \checkmark

And so
$$x = vt \Rightarrow t = \frac{x}{v} = \frac{0.020}{3.197 \times 10^6} \approx 6.3 \times 10^{-9} \text{ s.}$$

c The vertical distance covered is $\gamma = \frac{1}{2}at^2 \Rightarrow a = \frac{2\gamma}{t^2} = \frac{2 \times 0.25 \times 10^{-2}}{(6.3 \times 10^{-9})^2} \approx 1.3 \times 10^{14} \text{ m s}^{-2}$.

And from qE = ma we find $E = \frac{ma}{q} = \frac{9.11 \times 10^{-31} \times 1.3 \times 10^{14}}{1.6 \times 10^{-19}} \approx 740 \text{ N C}^{-1}$.

d The vertical component of velocity at B is $v_{\gamma} = at = 1.3 \times 10^{14} \times 6.3 \times 10^{-9} \approx 8.2 \times 10^{5} \text{ m s}^{-1}$.

Hence
$$\theta = \tan^{-1} \frac{v_{\gamma}}{v_x} = \tan^{-1} \frac{8.2 \times 10^5}{3.2 \times 10^6} \approx 14^\circ.$$

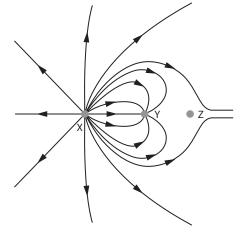
e The work done is the change in kinetic energy. \checkmark

Which is
$$\Delta E_{\rm K} = \frac{1}{2}mv_{\gamma}^2 = \frac{1}{2} \times 9.11 \times 10^{-31} \times (8.2 \times 10^5)^2 = 6.3 \times 10^{-17} \,\text{J}.$$

- **f** The work done is also $W = q\Delta V$ and so $\Delta V = \frac{W}{q} = \frac{6.3 \times 10^{-19}}{1.6 \times 10^{-19}} = 394 \approx 390 \text{ V}. \checkmark$
- **15 a** Field lines are mathematical lines originating and ending in electric charges. ✓
 Tangents to these lines give the direction of the electric field at a point. ✓
 - b They leave from positive charges (or infinity) and end in negative charges (or infinity). ✓
 They cannot cross. ✓

Their density is proportional to the electric field strength. \checkmark

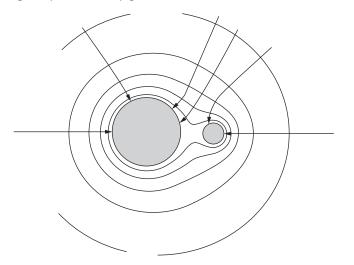
- **c** X is positive and Y is negative. \checkmark
- **d** i The field is zero at a position that may be approximated by Z. \checkmark



ii The ratio of the distance of Z from X to the distance from Y is about 2.5. \checkmark

Hence from
$$0 = \frac{kQ_X}{r_1^2} - \frac{kQ_Y}{r_2^2}$$
 we find $\frac{Q_X}{Q_Y} = \frac{r_1^2}{r_2^2} = 2.5^2 \approx 6.$

- 16 a i An equipotential surface is the set of all points that have the same potential. \checkmark
 - b i Field lines normal to equipotentials. ✓
 And normal to spheres. ✓
 (plus symmetrically paced lines one the lower side)



ii The potential difference between A and B is $\Delta V = 2.0 \times 10^6 \text{ J kg}^{-1}$. And so the work done is $m\Delta V = 1500 \times 2.0 \times 10^6 = 3.0 \times 10^9 \text{ J}$.

iii
$$g \approx \frac{\Delta V}{\Delta r} \checkmark$$

 $g \approx \frac{10^6}{4.0 \times 10^6} = 0.25 \text{ N kg}^{-1} \checkmark$

 ${f iv}$ From a very large distance away the two bodies look like one point particle. \checkmark

And the equipotential surfaces of a single particle are spherical. \checkmark c The potential; lines shown correspond to two masses so they are defined by $-\frac{GM_1}{r_1} - \frac{GM_2}{r_2} = \text{constant}$, or just

$$-\frac{M_1}{r_1} - \frac{M_2}{r_2} = \text{constant.} \checkmark$$

Two positive charges or two negative charges would give equipotential lines defined by

$$-\frac{Q_1}{r_1} - \frac{Q_2}{r_2} = \text{constant.} \checkmark$$

And so would be the same as in the gravitational case. \checkmark

Topic 2

Where appropriate, $1 \checkmark = 1$ mark

1	D
2	С
3	С
4	D
5	А
6	D
7	D
8	А
•	0

9 C

10 A

11 a i The equation applies to straight line motion with acceleration g. Neither condition is satisfied here. ✓ **ii** This equation is the result of energy conservation so it does apply since there are no frictional forces present. ✓

b From
$$v = \sqrt{2gh}$$
 we find $h = \frac{v^2}{2g} = \frac{4.8^2}{2 \times 9.81} = 1.174 \approx 1.2 \text{ m.}$

c i The kinetic energy at B is
$$E = \frac{1}{2}mv^2 = \frac{1}{2} \times 25 \times 4.8^2 = 28.8 \text{ J}. \checkmark$$

The frictional force is $f = \mu_{\rm K}N = \mu_{\rm K}mg = 0.45 \times 25 \times 9.81 = 110.36$ N and so the work done by this force is the change in the kinetic energy of the block, and so $110.36 \times d = 28.8 \Rightarrow d = 0.261 \approx 0.26$ m.

ii The deceleration is $\frac{f}{\mu} = \frac{110.36}{25} = 4.41 \text{ m s}^{-2}, \checkmark$

and so $0 = 4.8 - 4.41 \times t$ giving 1.1 s for the time.

d The speed at B is independent of the mass. \checkmark

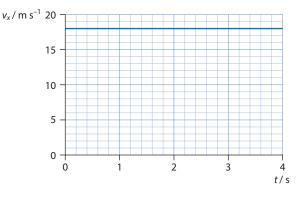
$$fd = \frac{1}{2}mv^2 \Rightarrow \mu_{\rm K}mgd = \frac{1}{2}mv^2 \Rightarrow d = \frac{v^2}{2\mu_{\rm K}}, \checkmark$$

and so the distance is also independent of the mass. \checkmark

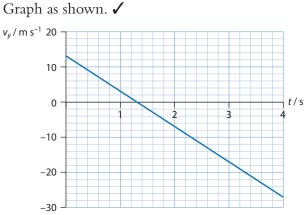
12 a i
$$v_x = v \cos \theta = 22 \times \cos 35^\circ = 18.0 \approx 18 \text{ m s}^{-1}$$

$$v_v = v \sin \theta = 22 \times \sin 35^\circ = 12.6 \approx 13 \text{ m s}^{-1} \checkmark$$

ii Graph as shown. ✓



1



b i At maximum height: $v_y^2 = 0 = u_y^2 - 2gy$.

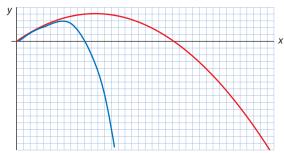
$$y = \frac{1}{2g} \checkmark$$

and so $y = \frac{12.6^2}{2 \times 9.8} = 8.1 \text{ m} \checkmark$
OR
 $v_y = 0 = v \sin \theta - gt \ 12.6 - 9.8t = 0 \checkmark$
so $t = 1.29 \text{ s} \checkmark$
Hence $y = 12.6 \times 1.29 - \frac{1}{2} \times 9.8 \times 1.29^2 = 8.1 \text{ m} \checkmark$

ii The force is the weight, i.e. $F = 0.20 \times 9.8 = 1.96 \approx 2.0$ N. \checkmark

c i
$$\frac{1}{2}mu^2 + mgh = \frac{1}{2}mv^2$$
 hence $v = \sqrt{u^2 + 2gh}$
 $v = \sqrt{u^2 + 2gh} = \sqrt{22^2 + 2 \times 9.8 \times 32} = 33.3 \approx 32 \text{ m s}^{-1}$
ii $v^2 = v_x^2 + v_y^2 \Rightarrow v_y = -\sqrt{v_x^2 - v_x^2} = -\sqrt{33.3^2 - 18.0^2} = -28.0 \text{ m s}^{-1}$
Now $v_y = u_y \sin\theta - gt$ so $-28.0 = 12.6 - 9.8 \times t$ hence $t = 4.1 \text{ s}$

d i Smaller height. ✓ Smaller range. ✓ Steeper impact angle. ✓



- ii The angle is steeper because the horizontal velocity component tends to become zero. ✓
 Whereas the vertical tends to attain terminal speed and so a constant value. ✓
- 13 a i In 1 second the mass of air that will move down is $\rho(\pi R^2 v)$.

The change of its momentum in this second is $\rho(\pi R^2 v)v = \rho \pi R^2 v^2$.

And from
$$F = \frac{\Delta p}{\Delta t}$$
 this is the force.

ii $\rho \pi R^2 v^2 = mg \checkmark$

And so
$$\nu = \sqrt{\frac{mg}{\rho \pi R^2}} = \sqrt{\frac{0.30 \times 9.8}{1.2 \times \pi \times 0.25^2}} = 3.53 \approx 3.5 \text{ m s}^{-1}.$$

- **b** The power is P = Fv where $F = \rho \pi R^2 v^2$ is the force pushing down on the air and so $P = \rho \pi R^2 v^2$. So $P = 1.2 \times \pi \times 0.25^2 \times 3.53^2 = 2.936 \approx 3.0 \text{ W}$
- **c** i From $F = \rho \pi R^2 v^2$ the force is now 4 times as large, i.e. 4mg and so the **net** force on the helicopter is 3mg.

And so the acceleration is constant at 3g. Hence
$$s = \frac{1}{2} \times 3g \times t^2 \Rightarrow t = \sqrt{\frac{2s}{3g}} \approx 0.90$$
 s.
ii $v = 3gt = \sqrt{\frac{2s}{3g}} \checkmark$
 $v \approx 26 \text{ m s}^{-1} \checkmark$

iii The work done by the rotor is $W = Fd = 4mgd = 4 \times 0.30 \times 9.8 \times 12 = 141$ J. \checkmark 14 a i The area is the impulse i.e. 2.0×10^3 N s. \checkmark

ii The average force is found from $\overline{F}\Delta t = 2.0 \times 10^3$ Ns. \checkmark

And so
$$\overline{F} = \frac{2.0 \times 10^{\circ}}{0.20} = 1.0 \times 10^{4} \text{ N. } \checkmark$$

Hence the average acceleration is $\overline{a} = \frac{1.0 \times 10^{4}}{8.0} = 1.25 \times 10^{3} \text{ m s}^{-2} \checkmark$

iii The final speed is $\overline{\nu} = \overline{at} = 1.25 \times 10^3 \times 0.20 = 250 \text{ m s}^{-1}$. And so the average speed is 125 m s⁻¹.

$$\mathbf{iv} \ s = \frac{1}{2} \overline{a}t^2 = \frac{1}{2} \times 1.25 \times 10^3 \times 0.20^2 \checkmark$$
$$s = 25 \text{ m} \checkmark$$

- **b** i The final speed is $\overline{\nu} = \overline{at} = 1.25 \times 10^3 \times 0.20$, \checkmark $\overline{\nu} = 250 \text{ m s}^{-1}$. \checkmark
 - ii The kinetic energy is $E_{\rm K} = \frac{1}{2}mv^2 = \frac{1}{2} \times 8.0 \times 250^2 \checkmark$ $E_{\rm K} = 2.5 \times 10^5 \text{ J} \checkmark$

iii
$$P = \frac{E_{\rm K}}{t} = \frac{2.5 \times 10^5}{0.20} \checkmark$$

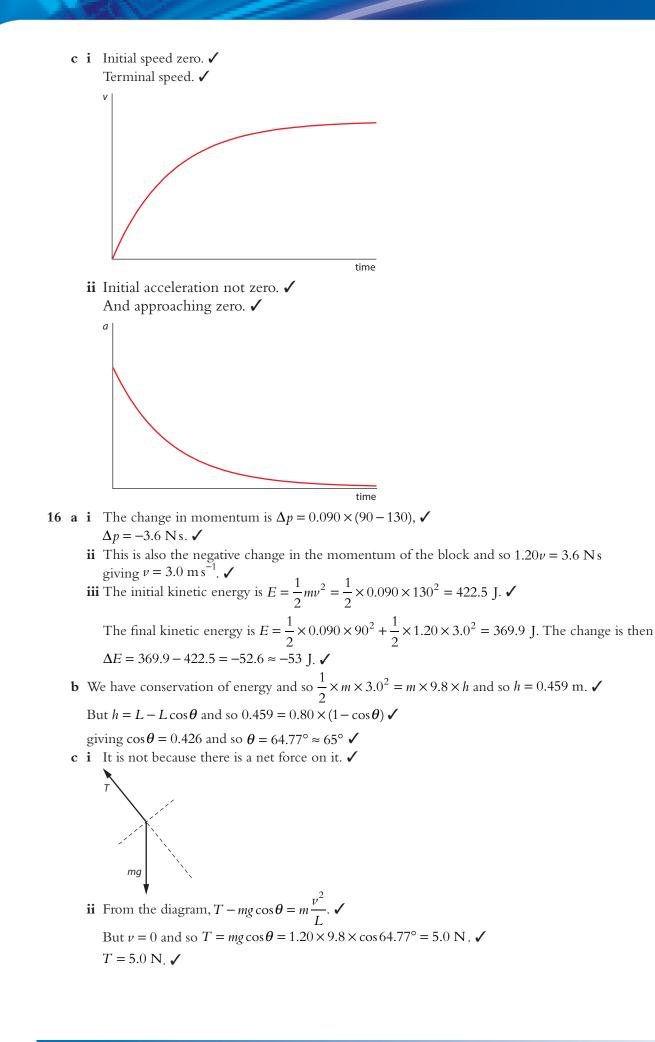
 $P = 1.25 \times 10^6 \text{ W }\checkmark$

- **15 a i** It is zero (because the velocity is constant). ✓
 - ii $F mg\sin\theta f = 0$
 - $F = mg\sin\theta + f = 1.4 \times 10^4 \times \sin 5.0^\circ + 600 \checkmark$

$$F = 1820 \text{ N} \checkmark$$

b The power used by the engine in pushing the car is $P = Fv = 1820 \times 6.2 = 1.13 \times 10^4$ W, $\checkmark P = 11.3$ kW. \checkmark

The efficiency is then $e = \frac{11.3}{15} = 0.75 \checkmark$



Topic 3

Where appropriate, $1 \checkmark = 1$ mark

- 1 A
- **2** B
- 3 C
- 4 D
- 5 B
- 6 A
- **7** B
- 8 A
- 9 D (the question should have specified equal moles for each gas)
- 10 A
- 11 a Use $pV = nRT \Rightarrow V = \frac{nRT}{n} \checkmark$

To find
$$V = \frac{1.0 \times 8.31 \times 273}{1.0 \times 10^5} = 2.27 \times 10^{-2} \,\mathrm{m}^3 \checkmark$$

b i There are $N_{\rm A} = 6.02 \times 10^{23}$ molecules.

So to each molecule corresponds a volume $\frac{2.27 \times 10^{-2}}{6.02 \times 10^{23}} = 3.77 \times 10^{-26} \text{ m}^3$.

ii Assuming a cube of this volume the side is $\sqrt[3]{3.77 \times 10^{-26}} = 3.35 \times 10^{-9}$ m, which is therefore an estimate of the separation of the molecules. \checkmark

This separation is much larger than the diameter of the helium atom and so the ideal gas approximation is good. \checkmark

c One mole of lead has a mass of 0.207 kg and a volume of $V = \frac{m}{\rho} = \frac{0.207}{11.3 \times 10^3} = 1.83 \times 10^{-5} \text{ m}^3$.

To each molecule corresponds a volume $\frac{1.83 \times 10^{-5}}{6.02 \times 10^{23}} = 3.04 \times 10^{-29} \text{ m}^3$.

Assuming a cube of this volume the side is $\sqrt[3]{3.04 \times 10^{-29}} = 3.12 \times 10^{-10}$ m which is therefore an estimate of the separation of the molecules.

d The ratio is then $\frac{3.35 \times 10^{-9}}{3.12 \times 10^{-10}}$, \checkmark

- 12 a Specific heat capacity is the amount of energy required to change the temperature of a 1 kg of a substance by 1 K. ✓
 - b One mole of any substance contains the same number of molecules; to raise the temperature by 1 K the internal energy will increase by the same amount and so the same heart must be provided. ✓
 One kg of different substances contains different numbers of molecules and so different amounts of energy are required to increase the temperature by 1 K. ✓

- c From $\frac{\Delta Q}{\Delta t} = \frac{\Delta m}{\Delta t} c \Delta T$ we find $600 = \frac{\Delta m}{\Delta t} \times 990 \times (40 20)$. So that $\frac{\Delta m}{\Delta t} = 3.0 \times 10^{-2} \text{ kg s}^{-1}$. d Then $\frac{\Delta V}{\Delta t} = \rho \frac{\Delta V}{\Delta t} = 1.25 \times 3.0 \times 10^{-2} = 3.8 \times 10^{-2} \text{ m}^3 \text{ s}^{-1}$.
- e The energy required is $Q = mL = 180 \times 2200 = 3.96 \times 10^5$ J. \checkmark

$$t = \frac{3.96 \times 10^5}{750} = 528 \,\mathrm{s} = 8.8 \,\mathrm{min}. \checkmark$$

13 a i The graph is a curve. ✓

If there was no air resistance the acceleration would have been constant and the velocity – time graph a straight line. \checkmark

ii We must estimate the area under the graph by counting squares with one small square equal in area to 0.5 m. ✓

There about 370 small squares so the height is about 185 m. \checkmark

iii Applying
$$mgh = \frac{1}{2}mv^2$$
 gives $v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 185}$, \checkmark
 $v = 60.2 \approx 60 \text{ m s}^{-1}$. \checkmark

b The impact speed is about 18.1 m s^{-1} implying a loss of mechanical energy of $\frac{1}{2} \times 8.0(60.2^2 - 18.1^2) = 1.32 \times 10^4 \text{ J}. \checkmark$

Assuming all of this goes into heating the ball and that this amount of energy warms the entire body uniformly. \checkmark $mc\Delta T = 1.32 \times 10^4$, \checkmark

and so
$$\Delta T = \frac{1.32 \times 10^4}{8.0 \times 320} \approx 5 \text{ K}. \checkmark$$

14 a The internal energy is the sum of the total random kinetic energy of the molecules and the intermolecular potential energy of the molecules of tungsten. ✓

b The tungsten loses heat 0.050 × 132 × (T − 31). ✓
 This heat is absorbed by the water and the calorimeter:
 0.300 × 4200 × (31 − 22) + 0.120 × 900 × (31 − 22) = 1.23 × 10⁴ J ✓

Hence $0.050 \times 132 \times (T - 31) = 1.23 \times 10^4$ or $T - 31 = \frac{1.23 \times 10^4}{0.050 \times 132} = 1864$ and finally $T = 1895 \approx 1900 \,^\circ\text{C}$.

c The calculated temperature is $T = \frac{Q}{m_W c_W} + 31$ where Q is the heat that went into the water and calorimeter. The actual Q would have been higher because some was transferred into the air during the move of the metal into the water.

Hence the calculated value is smaller than the actual temperature. \checkmark

- **15 a** The internal energy is the sum of the total random kinetic energy of the molecules and the intermolecular potential energy of the molecules of the substance. ✓
 - b During melting energy is supplied to the substance melting increasing its internal energy but not its temperature. ✓

Hence the student's statement is false. \checkmark

c The liquid is losing heat to the surroundings because the container is not insulated. ✓
 When the rate of heat loss is equal to the rate at which energy is being provided the temperature will remain constant. ✓

d The rate of heat loss is equal to the rate at which energy was being provided when the heater was on i.e. 35 W. ✓

Since
$$\frac{\Delta Q}{\Delta t} = mc \frac{\Delta T}{\Delta t}$$
 we have that $35 = 0.240 \times c \times \frac{3.1}{60}$.
And so $c = \frac{35 \times 60}{0.240 \times 3.1} = 2.8 \times 10^3$ J kg⁻¹ K⁻¹.
16 a $pV = nRT \Rightarrow n = \frac{pV}{RT}$ to find $n = \frac{250 \times 10^3 \times 1.50 \times 10^{-2}}{8.31 \times 273} = 1.653$.
So that $N_1 = nN_2 = 1.653 \times 6.02 \times 10^{23} = 9.95 \times 10^{23} \approx 1.0 \times 10^{24}$ molecules.

b As the tyre rolls on the road the rubber lining of the tyre expands and contracts generating thermal energy that heats the air in the tyre. ✓

The volume will increase.

And so will the pressure and temperature. \checkmark

c $p = \frac{nRT}{V} = \frac{1.653 \times 8.31 \times (273 + 35)}{1.60 \times 10^{-2}} = 2.64 \times 10^5$ Pa ≈ 260 kPa. ✓

d i Assuming the volume and temperature stay the same we must have that $\frac{p_1}{n_1} = \frac{p_2}{n_2}$ and so $\frac{250}{1.653} = \frac{230}{n_2}$ giving

 $n_2 = 1.52$. The number of molecules is then $N_2 = 1.52 \times 6.02 \times 10^{23} = 9.15 \times 10^{23}$.

The number of molecules that left is therefore $N_1 - N_2 = 9.95 \times 10^{23} - 9.15 \times 10^{23} = 8.0 \times 10^{22}$.

The rate of loss is then $\frac{8.0 \times 10^{22}}{8 \times 60 \times 60} = 2.8 \times 10^{18} \text{ s}^{-1}$.

ii The number of moles lost is 1.65 - 1.52 = 0.13 And so the lost mass of air is $0.13 \times 29 = 3.8$ g.

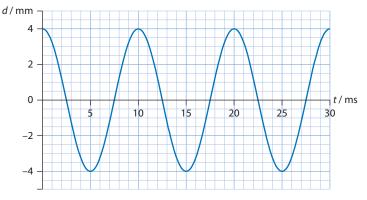
Topic 4

Where appropriate, $1 \checkmark = 1$ mark

- 1 A
- **2** C
- **3** B
- 4 A
- 5 D
- 6 D
- 7 D
- 8 C
- 0 0
- 9 B
- 10 A
- 11 a In a longitudinal wave the displacement is along the direction of energy transfer (DOET) ✓ whereas in a transverse wave it is at right angles to the DOET. ✓
 - **b** i The amplitude is 4.0 mm. \checkmark
 - ii The wavelength is 0.20 m. \checkmark
 - iii The period is 10 s and so the frequency is $f = \frac{1}{T} = \frac{1}{10} = 0.10$ Hz.
 - **c** The speed is $v = \lambda f = 0.20 \times 0.10$.

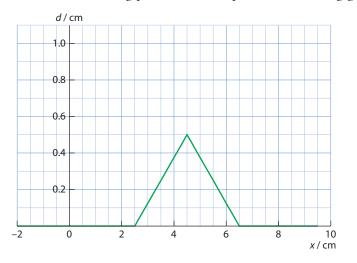
 $v = 0.020 \text{ m s}^{-1}$

- d Particle P has zero displacement at t = 10 s. ✓
 A short time later the displacement becomes positive (we look at the second graph). ✓
 To make the displacement of the point at 0.20 m positive a short time after 10 s the first graph must be shifted to the right, so the wave moves to the right. ✓
- e At t = 10 s point Q has displacement 4.0 mm. ✓
 Hence we must have the following graph. ✓



- **f** i The wavelength of the first harmonic is 4*L*, \checkmark and so 4*L* = 0.20 \Rightarrow *L* = 0.050 m. \checkmark
 - ii Standing waves do not transfer energy; travelling waves do. ✓
 Standing waves have variable amplitude; travelling waves have a constant amplitude. ✓
 - iii It is the speed of one of the travelling waves, ✓making up the standing wave. ✓

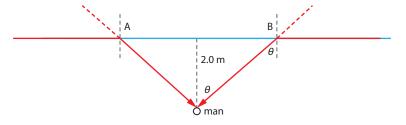
- 12 a When two waves (of the same type) meet, \checkmark
 - the resultant displacement is the algebraic sum of the individual displacements. \checkmark
 - **b** The speed of the black pulse is the same as that of the grey pulse since the medium is the same. \checkmark
 - **c** i The centres of the pulses are separated by a distance of 5.0 cm. The relative speed of the pulses is 30 m s⁻¹ and so will completely overlap at a time of $\frac{5.0}{30} = 0.167 \approx 0.17$ s.
 - ii In 0.167 s each pulse will move a distance of 2.5 m, ✓ and so the resulting pulse has the shape of the following graph. ✓



- d i The pulses have the same shape after the collision. ✓
 So no energy is lost (the collision of the pulses is elastic). ✓
 - ii The energy carried by a pulse is proportional to the (square of the) height of the pulse. ✓
 The pulse is short during overlap. ✓

But the string is moving vertically during overlap and so makes up for the apparently missing energy. \checkmark

13 a The diagram shows how rays of light coming in parallel to the water surface will refract. \checkmark



So the rays that can enter the man's eyes lie within a circle of diameter AB. \checkmark **b** From the diagram above and Snell's law $1.00 \times \sin 90^\circ = 1.33 \times \sin \theta$ so that $\theta = 48.8^\circ$. \checkmark

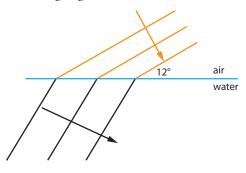
Hence $R = 2.0 \tan \theta = 2.0 \times \tan 48.8^\circ = 2.28 \approx 2.3 \text{ m}.$

c The angle θ will be the same. ✓
 But since the depth is greater so will the radius. ✓

d i Snell's law says that
$$\frac{\sin 12^\circ}{340} = \frac{\sin \theta}{1500}$$

so that $\theta = 66.5^{\circ} \approx 67^{\circ}$. \checkmark

 ii Three wavefronts as shown: Rays bending away from normal. ✓ Wavelength greater. ✓



iii The sound tends to move parallel to the surface of the water, ✓
 and not to penetrate deeper into the water where a swimmer might be. ✓

- 14 a Light in which the electric field oscillates on only one plane. \checkmark
 - **b** The intensity transmitted through the first polariser will be 160 W m⁻². \checkmark The intensity through the second will be $160 \cos^2 \theta$ W m⁻² and through the third $160 \cos^4 \theta$ W m⁻². \checkmark

Hence $160 \cos^4 \theta = 10$ giving $\theta = 60^\circ$.

c Let the intensities of the polarised and unpolarised components be $I_{\rm P}$, $I_{\rm U}$ respectively: at maximum transmitted intensity the polariser's axis will be parallel to the polarised light's electric field and the transmitted intensity will then be $I_{\rm P} + \frac{I_{\rm U}}{2}$; at minimum intensity the polarised component will not be transmitted and so the intensity will be $\frac{I_{\rm U}}{2}$.

We have that
$$\frac{I_{\rm P} + \frac{I_{\rm U}}{2}}{\frac{I_{\rm U}}{2}} = 7$$
 and so $\frac{I_{\rm P}}{I_{\rm U}} = 3$.

The required fraction is then $\frac{3}{4}$.

- d The wall is vertical and so the reflected light is partially polarised. ✓
 In a direction that is parallel to the wall, i.e. vertical. ✓
 And so a polariser with a horizontal transmission axis will cut off the reflected glare. ✓
- 15 a Light leaving each of the slits diffracts at each slit, ✓and so light from each slit will arrive at the middle of the screen. ✓
 - b With both slits open light arrives at the middle of the screen in phase and so the amplitude is twice the amplitude due to one slit. ✓
 The intensity is proportional to the amplitude squared. ✓
 So with one slit open the amplitude will be half and the intensity one quarter, i.e. 1 W m⁻². ✓
 - ${f c}$ The intensity of the side maxima is not the same as that of the central maximum. \checkmark
 - d The separation of the maxima on the screen is 0.60 cm and the separation is

given by
$$s = \frac{\lambda D}{d}$$
 and so $\lambda = \frac{sd}{D}$.
Hence $\lambda = \frac{0.60 \times 10^{-2} \times 0.39 \times 10^{-3}}{3.2} = 7.3 \times 10^{-7} \text{ m.}$

e Blue light has a smaller wavelength than red light. ✓ Hence the separation of the maxima will be less. ✓ 16 a A standing wave is formed when two identical travelling waves moving in opposite directions. ✓

Meet and superpose. \checkmark

- b i The travelling wave from the source reflects off the water surface. ✓
 The reflected wave superposes with the incoming wave creating a standing wave in the tube. ✓
 - ii The standing wave will have a wavelength equal to $\frac{4L}{n}$ where L is the length of the air column and n is an odd integer. \checkmark So for a given wavelength λ this will happen only when $L = \frac{\lambda n}{4}$, i.e. for specific values of the air column length. \checkmark
 - iii The difference in air column lengths is half a wavelength (explained in the next part) and so the next length is 37 cm. \checkmark

iv The difference in air column lengths is $\frac{\lambda n}{4} - \frac{\lambda(n-2)}{4} = \frac{\lambda}{2}$, i.e. half a wavelength and

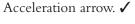
the wavelength is $\lambda = 2 \times 0.12 = 0.24 \text{ m} \cdot \checkmark$

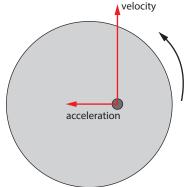
So $v = f\lambda = 1400 \times 0.24 = 336 \approx 340 \text{ m s}^{-1}$.

Topic 6

Where appropriate, $1 \checkmark = 1$ mark

- 1 A
- **2** C
- **3** B
- 4 C
- 5 C
- **J** (
- **6** B
- 7 D
- 8 D
- 9 C
- 10 A
- 11 a Velocity arrow. ✓





- **b** The angular speed is $\omega = \frac{2\pi}{1.40} = 4.488 \approx 4.5 \text{ rad s}^{-1}$.
 - The linear speed is $v = \omega r = 4.488 \times 0.22 = 0.987 \approx 0.99 \text{ m s}^{-1}$.
- **c** At maximum distance the frictional force will be the largest possible, i.e. $f_{\text{max}} = \mu_s N = \mu_s mg(= 0.434 \text{ N})$.

$$\mu_{s}mg = m\frac{v^{2}}{r} = m\frac{\omega^{2}r^{2}}{r}, \text{ hence } r = \frac{\mu_{s}g}{\omega^{2}} \checkmark$$
$$r = \frac{0.82 \times 9.8}{4.488^{2}} = 0.399 \approx 0.40 \text{ m }\checkmark$$

d i Using
$$r = \frac{\mu_s g}{\omega^2}$$
 we find $\omega = \sqrt{\frac{\mu_s g}{r}} \checkmark$
 $\omega = \sqrt{\frac{0.82 \times 9.8}{0.22}} = 6.0 \text{ rad s}^{-1} \checkmark$

ii The static frictional force can no longer supply the larger centripetal force required. ✓
 The body will then slide and the static frictional force is now replaced by the even smaller sliding frictional force; hence the disc will slide off the rotating platform. ✓

12 a From energy conservation: $\frac{1}{2}mv^2 = mgL$ so $v = \sqrt{2gL}$, ✓ $v = \sqrt{2 \times 9.8 \times 2.0} = 6.26 \approx 6.3 \text{ m s}^{-1}$. ✓ b $a = \frac{v^2}{L} = \frac{6.26^2}{2.0} = 19.6 \approx 20 \text{ m s}^{-2}$. ✓

c Weight vertically downwards. ✓ Larger arrow for tension upwards. ✓

d i A particle is in equilibrium if it moves with constant velocity. ✓ This particle moves on a circle and so cannot be in equilibrium. ✓

ii
$$T - mg = \frac{mv^2}{L} \checkmark$$

 $T = \frac{mv^2}{L} + mg = \frac{5.0 \times 6.26^2}{2.0} + 5.0 \times 9.8 = 147 \approx 150 \text{ N} \checkmark$
(or better: $T = \frac{mv^2}{L} + mg = \frac{m \times 2gL}{L} + mg = 3mg = 3 \times 5.0 \times 9.8 = 147 \approx 150 \text{ N}$)

13 a Correct arrows for tension. ✓Correct arrow for weight. ✓



b A particle is in equilibrium if it moves with constant velocity. ✓
 This particle moves on a circle and so cannot be in equilibrium. ✓

c i The vertical component of the tension equals the weight and so $T \cos \theta = mg$, i.e. $T = \frac{mg}{\cos \theta}$.

The horizontal component of the tension is $T\sin\theta$ and $T\sin\theta = m\frac{v^2}{r} = m\frac{v^2}{L\sin\theta}\checkmark$

Combining gives the answer $\nu = \sqrt{\frac{gL\sin^2\theta}{\cos\theta}}$.

ii The angular and linear speeds are related by $v = \omega r = \omega L \sin \theta$.

So
$$\omega = \frac{\sqrt{\frac{gL\sin^2\theta}{\cos\theta}}}{L\sin\theta}$$
.

Which is the answer $\omega = \sqrt{\frac{g}{L\cos\theta}}$.

d i
$$v = \sqrt{\frac{9.8 \times 0.45 \times \sin^2 60^\circ}{\cos 60^\circ}} = 2.57 \approx 2.6 \text{ m s}^{-1} \checkmark$$

ii $\theta = \sqrt{\frac{9.8}{0.45 \times \cos 60^\circ}} = 6.5997 \approx 6.6 \text{ rad s}^{-1} \checkmark$

- **e i** The air resistance force will reduce the speed of the ball. \checkmark
 - ii A graph of $\frac{\sin^2 \theta}{\cos \theta}$ shows that because the speed decreases, the angle will also decrease.

- iii The cosine of the angle will increase and hence the angular speed will decrease. \checkmark
 - (Note: These questions are best answered by considering the total energy of the ball:

$$E = \frac{1}{2}mv^{2} + mgh = \frac{1}{2}m\frac{gL\sin^{2}\theta}{\cos\theta} + mgL(1-\cos\theta) = \frac{1}{2}mgL\left(\frac{\sin^{2}\theta + 2\cos\theta - 2\cos^{2}\theta}{\cos\theta}\right)$$

The air resistance will reduce the total energy; graphing the total energy as a function of angle θ shows that for the energy to decrease the angle must decrease.)

14 a Measuring distances from the top of the sphere and using energy conservation shows that:

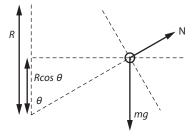
 $0 = \frac{1}{2}mv^2 - mgh$ where *h* is the vertical distance the marble falls.

From trigonometry: $h = R(1 - \cos \theta)$. \checkmark (see diagram that follows in **b**)

And so
$$0 = \frac{1}{2}mv^2 - mgR(1 - \cos\theta)$$
.

Manipulating gives $v = \sqrt{2gR(1 - \cos\theta)}$.

b The forces on the marble are the weight *mg* and the normal reaction force *N*:



Taking components of the weight gives $mg\cos\theta - N = \frac{mv^2}{R}$.

Hence
$$N = mg\cos\theta - \frac{mv^2}{R}$$
.

Substituting the expression for the speed from above gives $N = mg\cos\theta - 2mgR(1 - \cos\theta)$. And the result $N = mg(3\cos\theta - 2)$ follows.

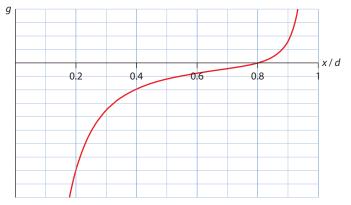
- **c** The marble will lose contact when $N \to 0$, i.e. when $\cos \theta = \frac{2}{3}$ or $\theta \approx 48^{\circ}$.
- **15 a** Calling this distance *x* we have that:

$$\frac{G16M}{x^2} = \frac{GM}{(d-x)^2} \checkmark$$
$$16(d-x)^2 = x^2 \text{ or } 4(d-x) = \pm x \checkmark$$

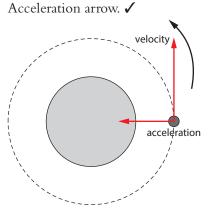
Only the plus sign gives a positive distance and so $x = \frac{4d}{5}$.

- $b\,$ Correct sign. $\checkmark\,$
 - Correct intersection. \checkmark

(The negative of this graph is also acceptable)



- **c i** The force is zero. ✓
 - ii The force from the larger mass will be larger because the particle will be closer to it. \checkmark Hence the net force will be directed towards the large mass. \checkmark
- **d** It will move to the left. \checkmark
 - With increasing speed and increasing acceleration. \checkmark
- **16 a i** Velocity arrow. ✓



- ii Acceleration is the rate of change of the velocity vector. \checkmark Here the velocity vector is changing because its direction is so we have acceleration. \checkmark
- **b** The force on the satellite is $\frac{GMm}{r^2} = m\frac{v^2}{r}$ i.e. $\frac{GM}{r} = v^2$.
 - Using $v = \omega r$, \checkmark
 - gives $\frac{GM}{r} = \omega^2 r^2$.

- From which the result $\omega^2 r^3 = GM$ follows. **c i** Since *r* decreases, from $\omega^2 r^3 = GM$ the angular speed will increase.
 - ii From $\frac{GM}{r} = v^2$, as *r* decrease *v* increases. $\omega^2 ... ^3$

d i Using
$$\omega^2 r^3 = GM$$
 we find $M = \frac{\omega}{G} \checkmark$
And so $M = \frac{(5.31 \times 10^{-5})^2 \times (2.38 \times 10^8)^3}{6.67 \times 10^{-11}} = 5.70 \times 10^{26}$ kg. \checkmark

ii Again using $\omega^2 r^3 = GM$ we find $\omega_T^2 r_T^3 = \omega_E^2 r_E^3$.

Hence
$$\omega_{\rm T} = \omega_{\rm E} \sqrt{\frac{r_{\rm E}^3}{r_{\rm T}^3}} = 5.31 \times 10^{-5} \times \sqrt{\left(\frac{2.38 \times 10^8}{1.22 \times 10^9}\right)^3} = 4.58 \times 10^{-6} \text{ rad s}^{-1} \checkmark$$

Hence
$$T = \frac{2\pi}{\omega_{\rm T}} = \frac{2\pi}{4.58 \times 10^{-6}} = 1.37 \times 10^6 \text{ s} = \frac{1.37 \times 10^6}{24 \times 3600} \text{ d} = 15.856 \approx 15.9 \text{ d}$$